CP symmetry and the strong interactions

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Abstract

I discuss several aspects of CP non-invariance in the strongly interacting theory of quarks and gluons. I use a simple effective Lagrangian technique to map out the region of quark masses where CP symmetry is spontaneously broken. I then turn to the possible explicit CP violation arising from a complex quark mass. After summarizing the definition of the renormalized theory as a limit, I argue that attempts to remove the CP violation by making the lightest quark mass vanish are not well defined. I close with some warnings for lattice simulations.

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I. INTRODUCTION

The SU(3) non-Abelian gauge theory of the strong interactions is quite remarkable in that, once an arbitrary overall scale is fixed, the only parameters are the quark masses. Using only a few pseudoscalar meson masses to fix these parameters, the non-Abelian gauge theory describing quark confining dynamics is unique. It has been known for some time [1] that, as the parameters are varied from their physical values, exotic phenomena can occur, including spontaneous breakdown of CP symmetry.

The possibility of a spontaneous CP violation is most easily demonstrated in terms of an effective chiral Lagrangian. In Section II I will review this model for the strong interactions with three quarks, namely the up, down, and strange quarks. This lays the groundwork for the discussion in Section III of where the CP violating phase arises. Section IV discusses how heavier states, most particularly the η' meson, enter without qualitatively changing the structure.

Included among the mass parameters of the strong interactions is a complex phase which, if present, explicitly violates CP symmetry. This parameter appears to be extremely small [2] since no such violation is seen phenomenologically. A puzzle for grand unification asks why is CP violation small for the strong interactions but not the weak [3]. It is sometimes suggested that a massless up quark would solve this problem, and I turn to this issue in section V. There I argue that asking whether the up quark mass vanishes is not physically meaningful. For this I elucidate the meaning of the continuum theory and the corresponding ambiguities in defining the quark mass. These issues remain even with the recently discovered chirally symmetric lattice fermions. Finally, Section VI contains some brief remarks, including possible impacts of the CP violating structures on lattice gauge simulations.

II. THE EFFECTIVE MODEL

A CP violating phase appears naturally in the simplest chiral sigma model of interacting pseudoscalar mesons. In this section I review the basic model and the standard connections between the quark masses and pseudoscalar meson masses. Nothing in this section is new; I am setting the stage for later discussion.

To be specific, consider the three flavor theory with its approximate SU(3) symmetry. Using three flavors simplifies the discussion, although the CP violating phase can also be demonstrated for the two flavor theory following the discussion in [4]. I work with the familiar octet of light pseudoscalar mesons π_α with $\alpha = 1\ldots8$. In a standard way (see for example [5]) I consider an effective field theory defined in terms of the SU(3) valued group element

$$\Sigma = \exp(i\pi_\alpha\lambda_\alpha/f_\pi) \in SU(3) \tag{1}$$

Here the λ_α are the usual Gell-Mann matrices which generate the flavor group and f_π is a dimensional constant with a phenomenological value of about 93 MeV. I follow the normalization convention that $\mathrm{Tr}\lambda_\alpha\lambda_\beta = 2\delta_{\alpha\beta}$. The neutral pion and the eta meson will play a special role in the later discussion; they are the coefficients of the commuting generators

$$\lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{2}$$

and

$$\lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}, \tag{3}$$

respectively. In the chiral limit of vanishing quark masses, we model the interactions of the eight massless Goldstone bosons with the effective Lagrangian density

$$L_0 = \frac{f_\pi^2}{4}\mathrm{Tr}(\partial_\mu\Sigma^\dagger\partial_\mu\Sigma) \tag{4}$$

The non-linear constraint of Σ onto the group SU(3) makes this theory non-renormalizable. It is to be understood only as the starting point for an expansion of particle interactions in powers of their momenta. Expanding Eq. (4) to second order in the meson fields gives the conventional kinetic terms for our eight mesons.

This theory is invariant under parity and charge conjugation. These operators are represented by simple transformations

$$\begin{aligned} P:\ \Sigma &\to \Sigma^{-1} \\ CP:\ \Sigma &\to \Sigma^* \end{aligned} \tag{5}$$

where the operation $*$ refers to complex conjugation. The eight meson fields are pseudoscalars. The neutral pion and the eta meson are both even under charge conjugation.

With massless quarks, the underlying quark-gluon theory has a chiral symmetry under

$$\begin{aligned} \psi_L &\to \psi_L g_L \\ \psi_R &\to \psi_R g_R \end{aligned} \tag{6}$$

Here (g_L, g_L) is in $(SU(3)\times SU(3))$ and $\psi_{L,R}$ represent the chiral components of the quark fields, with flavor indices understood. This symmetry is expected to be broken spontaneously to a vector SU(3) via a vacuum expectation value for $\overline{\psi}_L\psi_R$. This motivates the sigma model through the identification

$$\langle 0|\overline{\psi}_L\psi_R|0\rangle \leftrightarrow v\Sigma \tag{7}$$

The quantity v, of dimension mass cubed, characterizes the strength of the spontaneous breaking of this symmetry. Thus our effective field transforms under the chiral symmetry as

$$\Sigma \to g_L^\dagger \Sigma g_R \tag{8}$$

Our initial Lagrangian density is the simplest non-trivial expression invariant under this symmetry.

The quark masses break the chiral symmetry explicitly. From the analogy in Eq. (7), these are introduced through a 3 by 3 mass matrix M appearing in a potential term added to the Lagrangian density

$$L = L_0 - v\mathrm{Re}\ \mathrm{Tr}(\Sigma M) \tag{9}$$

Here v is the same dimensionful factor appearing in Eq. (7). The chiral symmetry of our starting theory shows the physical equivalence of a given mass matrix M with a rotated matrix $g_R^\dagger M g_L$. Using this freedom we can put the mass matrix into a standard form. I will assume it is diagonal with increasing eigenvalues

$$M = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} \tag{10}$$

representing the up, down, and strange quark masses. Note that this matrix has both singlet and octet parts under the vector flavor symmetry

$$M = \frac{m_u+m_d+m_s}{3} + \frac{m_u-m_d}{2}\ \lambda_3 + \frac{m_u+m_d-2m_s}{2\sqrt{3}}\ \lambda_8 \tag{11}$$

In general the mass matrix can still be complex. The chiral symmetry allows us to move phases between the masses, but the determinant of M is invariant and physically meaningful. Under charge conjugation the mass term would only be invariant if $M = M^*$. If $|M|$ is not real, then its phase is the famous CP violating parameter usually associated with topological structure in the gauge fields. For the moment I take all quark masses as real. Since I am looking for spontaneous CP violation, I consider the case where there is no explicit CP violation.

To lowest order the pseudoscalar meson masses appear on expanding the mass term quadratically in the meson fields. This generates an effective mass matrix for the eight mesons

$$\mathscr{M}_{\alpha\beta} \propto \text{Re Tr } \lambda_\alpha \lambda_\beta M \tag{12}$$

The isospin breaking up-down mass difference gives this matrix an off diagonal piece mixing the π_0 and the η

$$\mathscr{M}_{3,8} \propto m_u - m_d \tag{13}$$

The eigenvalues of this matrix give the standard mass relations

$$\begin{aligned}
m_{\pi_0}^2 &\propto \frac{2}{3}\left(m_u + m_d + m_s - \sqrt{m_u^2 + m_d^2 + m_s^2 - m_u m_d - m_u m_s - m_d m_s}\right) \\
m_{\pi_+}^2 &= m_{\pi_-}^2 \propto m_u + m_d \\
m_{K_+}^2 &= m_{K_-}^2 \propto m_u + m_s \\
m_{K_0}^2 &= m_{\overline{K}_0}^2 \propto m_d + m_s \\
m_\eta^2 &\propto \frac{2}{3}\left(m_u + m_d + m_s + \sqrt{m_u^2 + m_d^2 + m_s^2 - m_u m_d - m_u m_s - m_d m_s}\right)
\end{aligned} \tag{14}$$

Here I label the mesons with their conventional names.

Redundancies in these relations test the validity of the model. For example, comparing two expressions for the sum of the three quark masses

$$\frac{2(m_{\pi_+}^2 + m_{K_+}^2 + m_{K_0}^2)}{3(m_\eta^2 + m_{\pi_0}^2)} \sim 1.07 \tag{15}$$

suggests the symmetry should be good to a few percent. Further ratios of meson masses then give estimates for the ratios of the quark masses [5, 6, 7]. For one such combination, look at

$$\frac{m_u}{m_d} = \frac{m_{\pi^+}^2 + m_{K_+}^2 - m_{K_0}^2}{m_{\pi^+}^2 - m_{K_+}^2 + m_{K_0}^2} \sim 0.66 \tag{16}$$

This particular combination is polluted by electromagnetic effects; another combination partially cancels such while ignoring small $m_u m_d / m_s$ corrections

$$\frac{m_u}{m_d} = \frac{2m_{\pi^0}^2 - m_{\pi^0}^2 + m_{K_+}^2 - m_{K_0}^2}{m_{\pi^+}^2 - m_{K_+}^2 + m_{K_0}^2} \sim 0.55 \tag{17}$$

Later I will comment on a third combination for this ratio. For the strange quark, one can take

$$\frac{2m_s}{m_u + m_d} = \frac{m_{K_+}^2 + m_{K_0}^2 - m_{\pi^+}^2}{m_{\pi^+}^2} \sim 26 \tag{18}$$

III. SPONTANEOUS CP VIOLATION

So far all this is standard. Now I vary the quark masses and look for interesting phenomena. In particular, I want to find spontaneous breaking of the CP symmetry. Normally the Σ field fluctuates around the identity in SU(3). However, for some values of the quark masses this ceases to be true. When the vacuum expectation of Σ deviates from the identity, some of the meson fields will acquire expectation values. As they are pseudoscalars, this necessarily involves a breakdown of parity.

To explore this possibility, I concentrate on the lightest meson from Eq. (14), the π_0. From Eq. (14) we can calculate the product of the π_0 and η masses

$$m_{\pi_0}^2 m_\eta^2 \ \propto \ m_u m_d + m_u m_s + m_d m_s. \tag{19}$$

Whenever

$$m_u = \frac{-m_s m_d}{m_s + m_d} \tag{20}$$

the lowest order chiral relation gives a vanishing π_0 mass. For increasingly negative up-quark masses, our simple expansion around vanishing pseudoscalar meson fields fails. The vacuum is then no longer represented by Σ fluctuating around the unit matrix. Instead it fluctuates about an SU(3) matrix of form

$$\Sigma = \begin{pmatrix} e^{i\phi_1} & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & e^{-i\phi_1 - i\phi_2} \end{pmatrix} \tag{21}$$

where the phases satisfy

$$m_u \sin(\phi_1) = m_d \sin(\phi_2) = -m_s \sin(\phi_1 + \phi_2) \tag{22}$$

There are two minimum action solutions, differing by flipping the signs of these angles. The transition is a continuous one, with Σ going smoothly to the identity as the boundary given by Eq. (20) is approached.

In the new vacuum the pseudoscalar meson fields acquire expectation values. As the neutral pion is CP odd, we spontaneously break this symmetry. This will have various experimental consequences, for example eta decay into two pions becomes allowed since a virtual third pion can be absorbed by the vacuum. Fig. (1) shows the inferred phase diagram as a function of the up and down quark masses. Chiral rotations insure a symmetry under the flipping of the signs of both quark masses.

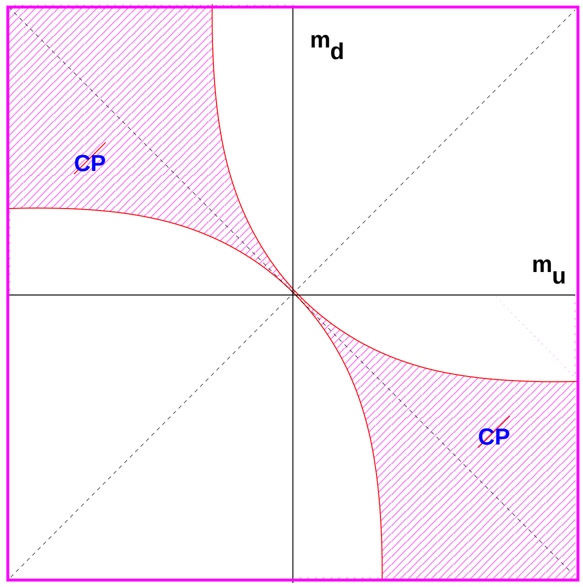

FIG. 1: The phase diagram of quark-gluon dynamics as a function of the two lightest quark masses. The shaded region exhibits spontaneous CP breaking. The diagonal lines with $m_u = \pm m_d$ trace where we have three degenerate pions due to isospin symmetry. The neutral pion mass vanishes on the boundary of the CP violating phase.

At first sight the appearance of such a phase at negative up quark mass seems surprising. Naively in perturbation theory the sign of a fermion mass can be rotated away by a redefinition $\psi \to \gamma_5 \psi$. However this rotation is anomalous, making the sign of the quark mass observable. A more general complex phase in the mass would also have physical consequences, i.e. explicit CP violation. With real quark masses the underlying Lagrangian is CP invariant, but there exists a large region where the ground state spontaneously breaks this symmetry.

Vafa and Witten [8] argued on rather general conditions that CP could not be spontaneously broken in the strong interactions. However their argument makes positivity assumptions on the path integral measure. When a quark mass is negative, the fermion determinant need not be positive for all gauge configurations; in this case the assumptions fail.

The possible existence of this phase was anticipated on the lattice some time ago by Aoki [10]. For the one flavor case he found this parity breaking phase with Wilson lattice gauge fermions. He went on to discuss also two flavors, finding both flavor and parity symmetry breaking. This case is now regarded as a lattice artifact of Wilson fermions. For a review of these issues see [11].

In conventional discussions of CP non-invariance in the strong interactions [9] appears a phase $e^{i\theta}$ appearing on tunneling between topologically distinct gauge field configurations. The famous

U(1) anomaly formally allows us to move this phase into the determinant of the quark mass matrix. After then rotating all phases into the up quark, we see that our spontaneous breaking of CP is occurring at an angle $\theta = \pi$. It does not, however, occur for up quark masses greater than a non-zero minimum value. There exists a finite region with $\theta = \pi$ that does not undergo this symmetry breaking. The chiral model indicates a smooth behavior in the quark mass when it is in the vicinity of zero. Indeed, from the effective Lagrangian point of view, the real and imaginary parts of the quark mass are completely independent parameters. The absence of experimental evidence for strong CP violation suggests that the imaginary part of the quark mass matrix vanishes, but says nothing about the real part.

An interesting special case occurs when the up and down quarks have the same magnitude but opposite sign for their masses, i.e. $m_u = -m_d$. In this situation it is illuminating to rotate the minus sign into the phase of the strange quark. Then the up and down quark are degenerate, and we have restored an exact vector $SU(2)$ flavor symmetry. The excitation spectrum will show three degenerate pions, but they will not be massless due to what might be thought of a vacuum condensate of eta particles.

IV. INCLUDING THE η'

The above discussion was entirely in terms of the light pseudoscalar mesons that become Goldstone bosons in the chiral limit. One might wonder how higher states can influence this phase structure. Of particular concern is the η' meson associated with the anomalous $U(1)$ symmetry present in the classical quark-gluon Lagrangian. Non-perturbative processes, including topologically non-trivial gauge field configurations, are well known to generate a mass for this particle. I will now argue that while this state can shift masses due to mixing with the lighter mesons, it does not make a qualitative difference in the existence of a phase with spontaneous CP violation.

The easiest way to introduce the η' into the effective theory is to promote the group element Σ to an element of $U(3)$ via an overall phase factor. Thus I generalize Eq. (1) to

$$\Sigma = \exp(i\pi_\alpha \lambda_\alpha / f_\pi + i\eta'/f_\pi) \in U(3) \tag{23}$$

Our starting kinetic Lagrangian in Eq. (4) would have this particle also be massless. One way to fix this deficiency is to mimic the anomaly with a term proportional to the determinant of σ

$$L_0 = \frac{f_\pi^2}{4} \mathrm{Tr}(\partial_\mu \Sigma^\dagger \partial_\mu \Sigma) - C|\Sigma|. \tag{24}$$

The parameter C parameterizes the strength of the anomaly in the $U(1)$ factor.

Now if we include the mass term exactly as before, additional mixing occurs between the η', the π_0, and the η. The corresponding mixing matrix takes the form

$$\begin{pmatrix} m_u+m_d & \frac{m_u-m_d}{\sqrt{3}} & m_u-m_d \\ \frac{m_u-m_d}{\sqrt{3}} & \frac{m_u+m_d+4m_s}{3} & \frac{m_u+m_d-2m_s}{\sqrt{3}} \\ m_u-m_d & \frac{m_u+m_d-2m_s}{\sqrt{3}} & m_a \end{pmatrix} \tag{25}$$

where m_a characterizes the contribution of the non-perturbative physics to the η' mass. This should have a value of order the strong interaction scale; in particular, it should be large compared to at least the up and down quark masses. The two by two matrix in the upper left of this expression is exactly what is diagonalized to find the neutral pion and eta masses in Eq. (14).

The boundary of the CP violating phase occurs where the determinant of this matrix vanishes. This modifies Eq. (19) to

$$m_{\pi_0}^2 m_\eta^2 m_{\eta'}^2 \ \propto\ M(m_u m_d + m_u m_s + m_d m_s) - m_u(m_d-m_s)^2 - m_d(m_u-m_s)^2 - m_s(m_u-m_d)^2 \tag{26}$$

The boundary shifts slightly from the earlier result, but still passes through the origin leaving Fig. (1) qualitatively unchanged.

V. CAN THE UP QUARK BE MASSLESS?

A oft proposed solution to the strong CP problem [12, 13, 14] asks whether $m_u = 0$. From the effective Lagrangian point of view, this appears to be an artificial setting of two parameters to zero, the real and imaginary parts of the quark mass. It is only the imaginary part that should vanish for CP to be a good symmetry, at least when the up quark mass is larger than the value giving spontaneous breaking.

While phenomenology, i.e. Eq. (17), seems to suggest that the up quark is not massless, there remains a lot of freedom in extracting that ratio from the pseudoscalar meson masses. From Eq. (14), the sum of the η and π_0 masses squared should be proportional to the sum of the three quark masses. Subtracting off the neutral kaon mass should leave just the up quark. Thus motivated, look at

$$\frac{m_u}{m_d} = \frac{3(m_\eta^2 + m_{\pi_0}^2)/2 - 2m_{K_0}^2}{m_{\pi^+}^2 - m_{K_+}^2 + m_{K_0}^2} \sim -0.8 \tag{27}$$

Thus even the sign of the up quark mass is ambiguous. Attempts to extend the naive quark mass ratio estimates to higher orders in the chiral expansion have shown that there are fundamental ambiguities in the definition of the quark masses [5]. But speculations on a vanishing up quark mass continue, so it is interesting to ask if this has physical meaning. In this section I investigate precisely what is meant by a quark mass, and what $m_u = 0$ would mean.

If two quark masses were to vanish simultaneously, then we would have exactly massless pions, Goldstone bosons for the resulting exact flavored chiral symmetry. Here I concentrate on whether the concept of a single massless quark has any meaning. While I could carry along the baggage of the heavier quarks, let me simplify the discussion and consider the theory reduced to a single flavor of quark. I will conclude that the question of whether m_u could vanish is not well posed.

Because renormalization is required, the concept of an "underlying basic Lagrangian" does not exist in quantum field theory. Instead there are some basic underlying symmetries, and the continuum theory is defined in terms of those and a few renormalized parameters. In practice this must be carried out as a limiting process on a cutoff version of the theory. As the lattice is the only well understood non-perturbative cutoff, it provides the most natural framework for such a definition. But any regulator must accomodate the known chiral anomalies, and thus there must be chiral symmetry breaking terms in the cutoff theory. These chiral breaking effects come in many guises. With a Pauli-Villars scheme, there is a heavy regulator field. With dimensional regularization the anomaly is hidden in the fermionic measure. For Wilson lattice gauge theory there is the famous Wilson term. With domain wall fermions there is a residual mass from a finite fifth dimension. With overlap fermions things are hidden in a combination of the measure and a certain non-uniqueness of the operator. I will return to this last case shortly.

The renormalization process tunes all relevant bare parameters as a function of the cutoff to fix a set of renormalized quantities. In the case of the strong interactions, the bare gauge coupling is driven to zero by asymptotic freedom. Its cutoff dependence is absorbed into an overall scale via the phenomenon of dimensional transmutation [15]. The only other parameters of the strong interactions are the quark masses. For these one inputs a few particle masses to finally determine the continuum theory uniquely. For the three flavor theory the most natural observables to fix these parameters are the pseudoscalar meson masses.

In the one flavor theory there are no Goldstone bosons, but massive mesons and baryons should exist. I need some physical parameter with which to carry out the renormalization of the quark mass. For this purpose I choose the ratio of the lightest boson mass to the lightest baryon mass. As both are expected to be stable, this precludes any ambiguity from particle widths. Calling the

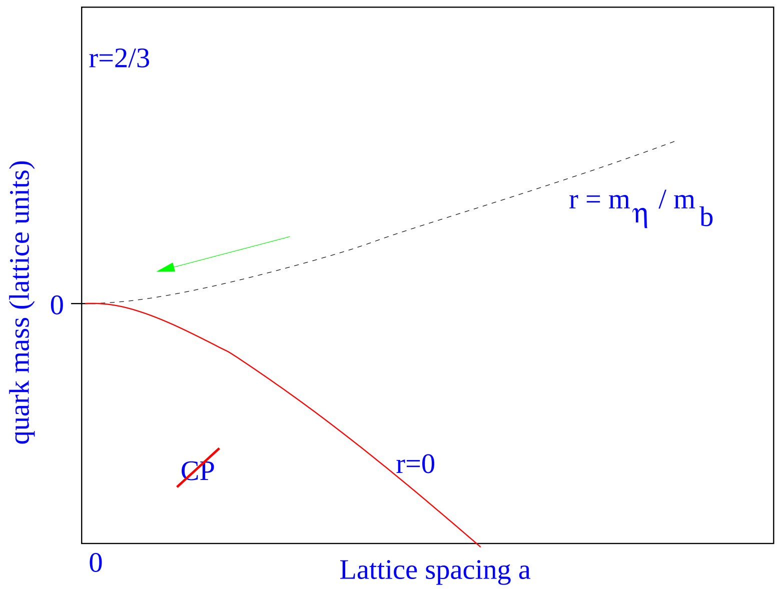

FIG. 2: Defining the continuum limit. For one flavor strong interactions I consider the ratio of the lightest boson to lightest baryon masses as my renormalized parameter. With the cutoff in place, we flow towards the origin along a curve of constant renormalized quantity. Below the contour where this ratio vanishes lies the region of spontaneous CP violation.

lightest boson the η and the baryon p I define

$$r = \frac{m_\eta}{m_p} \tag{28}$$

I expect to be able to adjust this parameter via the quark mass, which should be tuned to give the desired value. It should be possible to give this ratio any value throughout the range from $r = 0$ at the boundary of the above CP violating phase to $2/3$ in the heavy quark limit.

With the lattice cutoff in place, I can in principle determine this ratio given any values for the bare quark mass and bare coupling. For pedagogy, let me trade these parameters for the lattice spacing a and the quark mass in lattice units, $m_q a$. Both of these quantities go to zero in the continuum limit. The renormalization prescription is to select a desired value of r and follow the contour with this value in the (a, m_q) plane towards the origin. This process is sketched in Fig. (2). Perturbative divergences in the bare quark mass appear in the fact that these contours approach the origin with zero slope.

When the lattice spacing is non-zero, we expect artifacts to vary between different lattice prescriptions. In particular, the precise locations of the constant r contours will vary between different formulations. Holding the bare quark mass at zero will cross a variety of r contours, with none obviously favored as the origin is approached. Different cutoff schemes will give different continuum limits for $m_u = 0$, and thus asking that the up quark mass vanishes is physical.

With two or more degenerate flavors there will be one special contour where the lightest meson does represent a Goldstone boson. With the Wilson fermion formulation, the quark mass axis is represented by the hopping parameter. As this cutoff breaks chiral symmetry, the critical hopping parameter, where the meson mass vanishes, is renormalized away from its value in the continuum limit.

Recently there has been considerable progress with lattice fermion formulations that preserve a remnant of exact chiral symmetry [11]. With such, the two flavor theory will have the $r = 0$ contour preserved as the $m_q = 0$ axis. However, for the one flavor theory, this axis will still be expected to cut through various values of r. An interesting question is whether as we take the lattice spacing to zero along this axis, some physical value of r will be picked out as special and corresponding to vanishing quark mass. That this is unlikely follows from the non-uniqueness of these chiral lattice operators. For example, the overlap operator [16] is constructed by a projection process from the conventional Wilson lattice operator. The latter has a mass parameter which is to be chosen in a particular domain. On changing this parameter, we still have a good symmetry in the sense of the Ginsparg-Wilson relation [17], but the contours of constant r in Fig. (2) will be expected to shift around. Thus the horizontal axis is not expected to select one contour as special. Again, holding $m_u = 0$ is not expected to give a unique continuum theory.

VI. FINAL REMARKS

While I have been exploring rather unphysical regions in parameter space, these observations do raise some issues for practical lattice calculations of hadronic physics. Current simulations are done at relatively heavy values for the quark masses. This is because the known fermion algorithms tend to converge rather slowly at light quark masses. Extrapolations by several tens of MeV are needed to reach physical quark masses, and these extrapolations tend to be made in the context of chiral perturbation theory. The presence of a CP violating phase quite near the physical values for the quark masses indicates a strong variation in the vacuum state with a rather small change in the up quark mass; indeed, less than a 10 MeV change in the traditionally determined up quark mass can drastically change the low energy spectrum. Most simulations consider degenerate quarks, and chiral extrapolations so far have been quite successful. But some quantities, namely certain baryonic properties [18], do seem to require rather strong variations as the chiral limit is approached. These effects and the strong dependence on the up quark mass may be related.

Another issue is the validity of current simulation algorithms with non-degenerate quarks. With an even number of degenerate flavors the fermion determinant is positive and can contribute to a measure for Monte Carlo simulations. With light non-degenerate quarks the positivity of this determinant is not guaranteed. Indeed, the CP violation can occur only when the fermions contribute large phases to the path integral. Current algorithms for dealing with non-degenerate quarks take a root of the determinant with multiple flavors. In this process any possible phases are dropped. Such an algorithm is incapable of seeing any of the CP violating phenomena discussed here. This point may not be too serious in practice since the up and down quarks are nearly degenerate and the strange quark is fairly heavy. But these issues should serve as a warning that things might not work as well as we want.

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