BNL 35800

LATTICE OVERVIEW

Michael Creutz

Department of Physics
Brookhaven National Laboratory
Upton, New York 11973

Talk given at the annual meeting of the DPF of the APS, Sante Fe, NM, October 31 - November 3, 1984.

LATTICE OVERVIEW

Michael Creutz

Brookhaven National Laboratory,
Upton, New York 11973

ABSTRACT

After reviewing some recent developments in supercomputer access, I discuss a few areas where perturbation theory and lattice gauge simulations make contact. I conclude with a brief discussion of a deterministic dynamics for the Ising model. This may be useful for numerical studies of nonequilibrium phenomena.

SUPERCOMPUTER ACCESS

Monte Carlo calculations in lattice gauge theory have made theorists avid consumers of computer time. This need has often become difficult to meet, particularly at American universities with rather limited computational facilities. In order to investigate the severity of this problem, the High Energy Physics Advisory Panel (HEPAP) has recently commissioned two subpanels of which I am a member. The first was requested to study "the utilization and need for large scale computation for high energy physics theory research and to explore the benefits of access to a supercomputer." The second supbanel was asked for "recommendations relative to the automatic data processing resources needed during the next decade to maintain a forefront U.S. high energy physics program."

The theory panel, chaired by C. Quigg of Fermilab, presented its report at the HEPAP meeting at SLAC in September of this year. This report addressed the immediate short term needs of the community, leaving longer term issues to the other subpanel. The main conclusion of this report was that theoretical computing now needs the equivalent of three class VI machines. The dominant use is for lattice gauge calculations, which should occupy two thirds of this capacity. The majority of the remaining supercomputing time is needed for design studies related to the SSC.

In addition, the report noted the necessity for a good computing environment to efficiently utilize this capacity. In particular, university users will need good networks and access equipment. Finally the report emphasized that research in special purpose devices as well as array processors should be pursued as potential routes to vast amounts of economical computational power.

The second subpanel, chaired by J. Ballam of SLAC, will address both experimental and theoretical needs for the next decade. This group plans on presenting a report in March of 1985. Some interesting issues which need to be resolved are whether theorists can efficiently utilize the same machines as used for experimental data processing and whether the high energy physics community should set up one or more special centers to handle the massive amounts of computation anticipated in the future.

I should note that in the immediate future it appears that we will have a rather abrupt increase in the availability of supercomputing time. The MFE computing center at Livermore has added a CRAY XMP to its facility. This machine will be devoted to providing supercomputing to people doing energy research for the DOE. In particular, high energy physics should get several thousand hours of this time. The DOE is also installing a CYBER 205 at Florida State University. The theoretical community will hopefully become major users of this machine. Finally, the NSF has a computer initiative with 40 million dollars allocated for 1985. Of this, about 30 million dollars will be used to buy time on existing supercomputers for use by various research groups.

MONTE CARLO SIMULATION AND PERTURBATION THEORY

Let me now turn to physics. Lattice gauge theory has become our primary tool for the study of non-perturbative phenomena in the non-Abelian gauge theory of the strong interactions. Its successes in investigating long distance phenomena have nicely complemented the use of perturbation theory in the short distance domain. What I would like to do in this talk is mention a few cases where perturbation theory and Monte Carlo simulations have been able to make contact.

At a fairly trivial level, perturbation theory is useful for the initial testing of a Monte Carlo program. Indeed, the perturbative prediction for the expectation value of the action should be approached as the coupling becomes small. This plus a matching with a low order strong coupling expansion are essential for establishing confidence in any new program.

A slightly more sophisticated use of perturbation theory is for improving Monte Carlo renormalization group calculations. Two rather different approaches have been recently used for this purpose. The first is the "canonical" approach as developed by the "canons" Wilson and Kadanoff[1]. Here one starts with a theory on a fine lattice and attempts to integrate out some of the degrees of freedom to obtain an equivalent theory on a coarser lattice. For example, given a lattice partition function defined in terms of an action S(U)

$$Z = \int (dU)\, e^{-\beta S(U)} \tag{1}$$

we can define a renormalized action on a coarser lattice by integrating out some of the variables

$$e^{-\beta S_R(\bar{U})} = \int (dU)\, e^{-\beta S(U)} \prod_{\bar{U}} \delta(\bar{U} - K(U)) \tag{2}$$

The partition function is now given by an integral over the block variables $\bar{U}$

$$Z = \int (d\bar{U})\, e^{-\beta S_R(\bar{U})} \tag{3}$$

An example of a possible blocking is that of Swendsen where the variable $\bar{U}$ is chosen to be the group element closest to the sum over 7 paths of the product of the original lattice variables linking sites separated by two fundamental lattice units.[2]

Although equation (3) is exact, the renormalized action S depends in principle on an infinite number of couplings involving arbitrarily separated links. Therefore, in any practical calculation some truncation is necessary. The severity of this truncation will in general depend on the kernal K defining the block variables. It is here that perturbation theory can provide a useful guide. In particular, the blocking procedure should give a shift in the bare coupling only to order g_0^4. Recently Gocksch and Ogilvie have perturbatively studied the Swendsen procedure with a truncation down to a single coupling constant[3]. This gives an unwanted tree level shift in the bare coupling. They then suggested a modification of the Swendsen procedure, wherein those paths of length 4 are weighted less heavily than paths of length 2. They found that a factor of 0.125 in front of the longer paths made the tree level shift vanish.

A second technique for renormalization group studies is the ratio method presented in ref. (4). Here one constructs a dimensionless ratio of Wilson loops which should be finite in a continuum limit. In particular, it should be constructed so that all self energy divergences cancel. Such a ratio R will be a function of the size r of the loops used, the lattice spacing a, and the bare coupling g_0, which upon renormalization becomes itself a function of the cutoff. As R should be finite when a goes to zero, we should have

$$\begin{aligned} R(r,a,g_0(a)) &= R(r,a/2,g_0(a/2)) + O(a^2/r^2) \\ &= R(2r,a,g_0(a/2)) + O(a^2/r^2) \end{aligned} \tag{4}$$

The final step in this equation is dimensional analysis. Thus we see that by comparing such ratios measured on two length scales, we can determine how the bare coupling changes under a change of the cutoff by a factor of two.

The main approximation in this second renormalization group approach is the need to drop the order a^2/r^2 terms. In ref. (5) it has been argued that perturbation theory can be useful here as well. Those authors form linear combinations of physical ratios so that any tree level shift in the coupling coming from these correction terms will

cancel. They also suggest that one could further constrain these combinations so that the one loop shift will agree with the continuum perturbative result.

Let me now change the subject slightly and discuss the perturbative corrections to mass ratios calculated on a lattice of finite spacing. As should be familiar, in lattice gauge theory we are interested in measuring quantities such as a correlation length ξ on the lattice. The inverse of this quantity represents the mass of some particle in lattice units

$$\xi^{-1} = ma \tag{5}$$

Asymptotic freedom tells us how the lattice spacing is related to the bare coupling as we go to the continuum limit

$$a = (a/\Lambda_0)(g_0^2\beta_0)^{-\beta_1/2\beta_0^2}\, e^{-1/(2\beta_0 g_0^2)}(1 + O(g_0^2)) \tag{6}$$

where β_0 and β_1 are the first two coefficients in the perturbative expansion of the renormalization group function[6]. If we can measure how the correlation length diverges as the bare coupling goes to zero, then we are effectively measuring the particle mass in units of Λ_0, an integration constant of the renormalization group equation.

Since the initial measurements of the square root of the string tension via this technique, the numerical value has drifted by a factor of nearly two as the calculations improved. This has been blamed on the $(1+O(g_0^2))$ terms in Eq. (6) and the corresponding violation of asymptotic scaling. I would like to argue, however, that ratios of masses, such as the glueball mass to the square root of the string tension, should scale considerably better. Indeed, I will now review a well known argument that such mass ratios with a finite cutoff will differ from their continuum values by terms which go to zero faster than any power of the coupling

$$\begin{aligned} m_1(a)/m_2(a) &= m_1(0)/m_2(0)\ (1 + O(a^2/m^2)) \\ &= m_1(0)/m_2(0)\ (1 + O(e^{-1/(\beta_0 g_0^2)})). \end{aligned} \tag{7}$$

I will present the argument in conventional perturbative language, in terms of a renormalized coupling defined at some scale μ. Such a renormalized coupling can be perturbatively expanded as a power series in the bare coupling

$$g_R = g_R(\mu a, g_0) = \sum_n c_n(\mu a) g_0^n \tag{8}$$

As is well known, the coefficients in this series are logarithmically divergent as the cutoff a is taken to zero. A renormalization scheme at scale μ determines the bare coupling as a function of the lattice spacing by requiring that the renormalized coupling be fixed as a is varied. Inverting eq. (8) gives the bare coupling as a function of the renormalized one

$$g_0 = \sum_n \bar{c}_n(\mu a) g_R^n \tag{9}$$

We now wish to compare renormalizing the theory at two different scales, μ_1 and μ_2. In particular, how much does $g_R(\mu_2)$ vary if $g_R(\mu_1)$ is held fixed? Consider expanding the renormalized coupling at one scale in terms of the coupling at the other

$$\begin{aligned} g_R(\mu_2 a, g_0) &= \sum_n d_n g_R^n(\mu_1 a, g_0) \\ &= \sum_n c_n(\mu_2 a)\Big(\sum_m \bar{c}_m(\mu_1 a) g_R^m(\mu_1 a, g_0)\Big)^n \end{aligned} \tag{10}$$

Since both renormalized couplings are finite in the continuum limit, all divergences must cancel from the coefficients d_n. Furthermore, the coefficients c_n and $\bar{c}_n$ cannot have any inverse logarithms. Thus any finite cutoff variation of the d_n must take the form

$$d_n(\mu_1/\mu_2, \mu_1 a) = d_n(\mu_1/\mu_2, 0) + O(a^2\mu_1^2) \tag{11}$$

This means that if $g(\mu_1)$ is held fixed, then $g_R(\mu_2)$ will be constant up to terms of order a^2, which is of order $\exp(-1/(\beta_0 g_0^2))$. Now a possible non-perturbative renormalization scheme would be to hold some physical particle mass fixed. Extrapolating the above result to such a scheme, we reach our conclusion that any other mass will be fixed up to terms which vanish faster than any power of the bare coupling.

As a final point of contact between perturbation theory and lattice results, let me describe a

recent comparison of the asymptotic freedom scales for SO(3) and SU(2) lattice gauge theories[7]. These theories both have the same SU(2) Yang-Mills theory for their naive classical limit, and thus their comparison is essentially a test of universality of the continuum limit. This particular comparison is interesting for several reasons. First, the models differ in their phase structure at finite cutoff; in particular the SO(3) model exhibits a first order phase transition[8], while the SU(2) one has no transition. Second, the fundamental representation Wilson loops vanish trivially for SO(3), which does not recognize the group center. This means that a string tension is difficult to define and it is simpler to use other quantities for matching purposes. Finally, the SO(3) model has "monopoles" of low action when the cutoff is finite. These should be suppressed in the continuum limit, but are presumably the source of the SO(3) phase transition[9].

If the continuum limit is indeed universal, then these two cutoff schemes should differ at most in the scale parameter, which we denote by Λ_F or Λ_A for the SU(2) or SO(3) formulations, respectively. The ratio of the corresponding parameters follows from a one loop calculation, which gives[10]

$$\Lambda_F/\Lambda_A = 28.9 \qquad (12)$$

The goal of ref. (7) was to verify this perturbative result using Monte Carlo simulation. In addition to the above points, this calculation is technically interesting for several reasons. First, the numerical value in eq. (12) is rather large, particularly in comparison with other checks of universality. Indeed, SO(3) is a rather large variation on the standard SU(2) model. Secondly, because the fundamental loops vanish trivially for SO(3), we are forced to use adjoint loops in our comparison. This shows that one can use more abstract physical quantities than the string tension for such calculations. Third, we must work on the weak coupling side of the SO(3) phase transition, which occurs at a rather small value of the bare coupling constant. We will be matching physical quantities when the SU(2) coupling $\beta = 4/g_0^2$ is in the vicinity of 10. If we use previous values of the string tension to normalize our scale and set this to its physical SU(3) value, then we are working with lattice spacings on the order of 10^{-22} cm. At such a lattice spacing any string tension would be extremely small and the correlation length is much larger than the lattice; nevertheless, we obtain some physically meaningful results. Finally, even though the coupling is rather small, this extreme variation on the lattice action gives rise to rather large two loop corrections

Our procedure was to form a ratio of loops which cancels any ultraviolet divergences. From this we define a renormalized coupling which we then match between the two models. For example, in fig. 1 we show the renormalized coupling calculated from the ratio

$$R_2 = \frac{W(2,2)\ W(1,1)}{(W(1,2))^2} \qquad (13)$$

where W(I,J) represents an adjoint Wilson loop of size I by J. By plotting the inverse of the respective couplings, the one loop change in bare couplings becomes a shift in the respective curves. Measuring this shift at various couplings, we obtain a raw value for the ratio of Λ_F/Λ_A which ranges from 30 to 90 depending on the value of the bare coupling used in the comparison. Applying the two loop correction as calculated in ref. (11), this number becomes more stable with a value ranging from 21 to 34, which we regard as quite acceptable agreement with eq. (12).

In our calculations we also tried matching single Wilson loops directly. Although these have naive perturbative divergences, with both actions we are working with the same value for the cutoff and thus the divergences should match to lowest order. This matching will persist to one loop because the basic diagram which shifts the divergent part is the same one that gives the shift in the finite part of the loop. We do not know if this matching of the loops themselves is justified to still higher order, although naively applying the two loop corrections of ref. (12) gives excellent agreement with the ratio of Λ_F/Λ_A as calculated above.

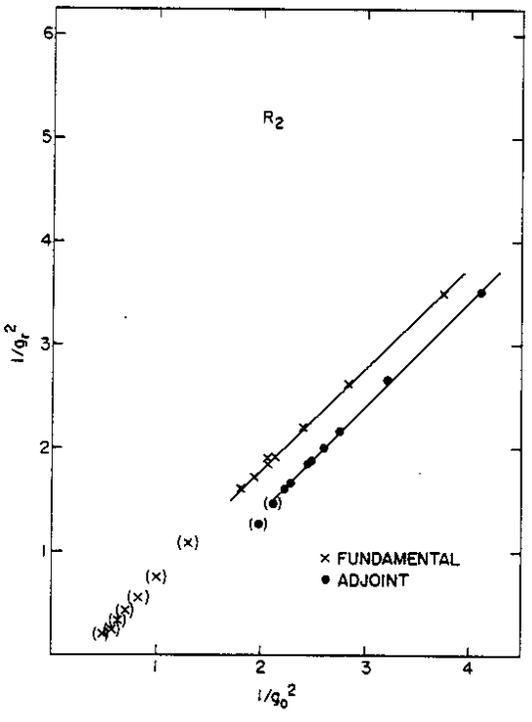

FIG. 1. The inverse renormalized coupling at a scale of 2 lattice spacings as calculated from the ratio in eq. 13. This is given as a function of the inverse bare coupling for both fundamental and adjoint actions.

DETERMINISTIC ISING DYNAMICS

Let me now sharply change the subject and conclude this talk with a brief mention of a model which I am currently developing. This is a deterministic cellular automaton rule which will simulate the Ising model. The idea is to formulate a simple deterministic dynamics for the numerical study of nonequilibrium phenomena such as relaxation and heat flow. The algorithm is a generalization of the microcanonical Monte Carlo method discussed in ref. (12). However in this case the "demons" of that reference do not move around, but are attached to the Ising sites and play the role of a momentum conjugate to the spins. The updating is done in a checkerboard style to avoid simultaneous flipping of nearest neighbors.

The procedure differs conceptually from the dynamics presented by Glauber[13] in that the evolution of a given configuration is purely deterministic. Also we have a locally defined energy which is precisely conserved. The temperature is not a parameter defining the dynamics, but is a statistical concept, defined only by averages over space or time or both.

The method has the practical advantage that all operations on the spins and the momentum variables are simple bit manipulations. Thus the algorithm can use such techniques as multi-spin coding to run extremely fast on conventional computers. Indeed, it is capable of simulating the heat equation, a rather generic partial differential equation, without using any floating point arithmetic. It would be interesting to see if such a technique is generally a more efficient approach to solving such equations than conventional discretizations.

ACKNOWLEDGMENT

This work was supported by the Department of Energy Contract No. DE-AC02-76CH00016.

REFERENCES

1. K. Wilson, Phys. Rev. B4, 3174 (1971); B4, 3184 (1971);
L. Kadanoff, Ann. Phys. 100, 359 (1976).
2. R. H. Swendsen, Phys. Rev. Lett. 47, 1775 (1981).
3. A. Gocksch and M. Ogilvie, private communication.
4. M. Creutz, Phys. Rev. D23, 1815 (1981).
5. A. Hasenfratz, P. Hasenfratz, U. Heller, and F. Karsch, Phys. Lett. 140B, 76 (1984).
6. H. D. Politzer, Phys. Rev. Lett. 30, 1343 (1973);
D. Gross and F. Wilczek, Phys. Rev. Lett. 30, 1346 (1973); Phys. Rev. D8, 3633 (1973);
W. E. Caswell, Phys. Rev. Lett. 33, 244 (1974);
D. R. T. Jones, Nucl. Phys. B75, 531 (1974).
press).
8. J. Greensite and B. Lautrup, Phys. Rev. Lett. 47, 9 (1981);
I. G. Halliday and A. Schwimmer, Phys. Lett. 101B, 327 (1981).
9. I. G. Halliday and A. Schwimmer, Phys. Lett. 102B, 337 (1981).
10. A. Gonzales-Arroyo and C. P. Korthals Altes, Nucl. Phys. B205 [FS5], 46 (1982).
11. J. Jurkiewicz, C. P. Korthals Altes and J.W. Dash, Nucl. Phys. B233, 457 (1984);
R. K. Ellis and G. Martinelli, Nucl. Phys. B235 [FS11], 93 (1984).
12. M. Creutz, Phys. Rev. Lett. 50, 1411 (19840; Proceedings of the Argonne National Laboratory Workshop on Gauge Theory on a Lattice (Argonne Nat. Lab., 1984).
13. R.J. Glauber, J. Math. Phys. 4, 294 (1963).