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EXTENDED OBJECTS†

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After some disconnected comments on the MIT bag and string models for extended hadrons, I review current understanding of extended objects in classical conventional relativistic field theories and their quantum mechanical interpretation.

In recent years it has become increasingly popular to study classical theories containing extended particle-like objects as a starting point for formulating a quantum theory of relativistic particles. Several motivations for this are

1) One gets around the standard perturbation approach with a fundamental field for each particle.
2) Since the proton radius is of order a fraction of a Fermi, perhaps one should start with a theory where the lowest order radius is not zero.

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3) Regge recurrences and daughters arise naturally as excitations of normal modes of extended objects.

4) The phenomenological successes of the MIT bag model[1] encourage further study of other extended models.

The extended objects studied so far seem to fall into three main classes:

1) Theories of extended objects put in by hand at the outset, i.e., the MIT bag and the string models.

2) Conventional local interacting field theories with non-dissipating classical solutions.

3) "Old fashioned" bound states, i.e., the hydrogen atom, the deuteron, and perhaps charmonium.

In this talk I will concentrate primarily on reviewing what is known about extended objects of the second type. Before this, however, I will make a couple of disconnected remarks on the first type.

Although the fundamental objects are extended, nonetheless the MIT bag and the usual string models are local theories. By this I mean that the classical equations of motion determine the time derivatives of local quantities in terms of other local quantities at the point in question. In physical terms, a kick on one side of a bag will not be felt by the other until a signal has been propagated in a local manner either through the bag interior or along the bag surface.

A recent paper by Bars[2] has shown an equivalence in two-dimensional space-time between a string model and a local theory of quarks and non-Abelian gauge fields. This is particularly intriguing in that it may extend to higher dimensions. Indeed, Wilson's[3] Feynman rules for strongly coupled gauge fields on a lattice strongly resemble a path integral formulation of a string theory. Also, 't Hooft[4] has argued that for large N in the gauge group SU(N), the dominant diagrams in a gauge theory have a simple stringlike topology.

I now return to conventional local field theory and discuss a phenomenon that has been given many names, i.e., extended object, lump, soliton, non-dissipative solution, kink, extremeon, etc. The term soliton has become the most accepted so I shall also use it. A soliton is a sort of ball lightning, a solution to a classical field theory possessing a localized energy concentration that holds itself together as time evolves. It is distinguished from a conventional wave packet by the requirement that it not spread. This definition is less restrictive than that used by the applied mathematicians,[5] who also require that the soliton remain unchanged in collisions. In particle physics, where we expect collisions to excite new states, we have no particular reason to impose this more restrictive condition.

Many examples of solitons are now known in relativistic field theories. The most studied are the topological solitons, exemplified by the sine-Gordon equation in two dimensional space time. The Lagrangian density for this theory is

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - (1 + \cos\beta\phi) \qquad (1)$$

and the static soliton solution is

$$\phi(x) = \frac{2}{\beta} \arctan(\sinh\beta\phi) \qquad (2)$$

This equation can be crudely approximated by a length of ribbon lying on a table. The field variable is represented by the transverse angle of the ribbon with respect to the table, and the soliton solution is a half twist of the ribbon. In this model there exist both solitons and anti-solitons corresponding to right and left handed twists. The 't Hooft Polyakov monopole[6] generalizes the concept of a topological soliton to four-dimensional space-time. The stability of topological solitons is related to the possible topologically distinct mappings of spatial infinity onto the manifold of allowed vacuum values for a set of Higg's fields.[7] In four dimensions, topological solitons of finite energy require the introduction of gauge fields.

Another extensively studied class of solitons uses a charge carrying field to displace a second field coherently from its normal vacuum value. This second field then acts back on the charged field to keep the charge distribution localized.[8,9] These solitons can also be crudely demonstrated by a ribbon, where a weight (with conserved baryon number) can hold a small section of the ribbon in an inverted configuration with half twists on either side. The presence of the weight keeps the half twists from annihilating. This type of soliton can be constructed from scalar fields alone, but is then necessarily time dependent. This time dependence can be a trivial time dependent phase for the charged field. Friedberg, Lee and Sirlin[9] showed that these theories have two critical values for the soliton charge, a lower one Q_C below which the soliton ceases to be stable under small perturbations and a larger Q_S above which the soliton is absolutely stable. In between the soliton is metastable; large fluctuations will allow it to decay.

In addition to these two main classes of solitons, a few others are known. In the sine-Gordon theory in two dimensions a solution called the "breather" consists of a bound soliton-antisoliton pair passing back and forth through each other. This periodic solution is analytically known and satisfies our definition for a soliton. Theories with no particular symmetry can possess solitons. For example consider a complex field in two dimensional space-time with Hamiltonian density

$$\mathcal{H} = |\dot{\phi}|^2 + |\partial_x \phi|^2 + (|\phi|^2 - f^2)^2 - C(\phi + \phi^+) \qquad (3)$$

For small but non-vanishing C, this theory has a soliton solution where at $x = \pm\infty$, ϕ is approximately f but as we pass from $x = -\infty$ to $x = +\infty$ the phase of ϕ goes from zero to 2π. Reexpressing the field in terms of its phase and magnitude, this looks much like a topological soliton; however, it differs in that the solution is only metastable.

Let me emphasize that outside the topological and charge carrying solitons, almost nothing is known. There may exist many solitons waiting to be found. These will be difficult to find without some new intuition to guide us.

In the past couple of years a great deal of effort has gone into finding the meaning of these solitons in quantized field theory. The answer is that they are particles, a new class of heavy particles that do not appear in a conventional perturbative approach.[10] For small nonlinearity in the field theory, the quantization proceeds as a perturbation theory in quantum fluctuations about the classical soliton. In this way one finds states for the soliton that are labelled by momentum and occupation numbers for the discrete normal modes of vibration

$$|P; n_1, \ldots, n_m> \qquad (4)$$

Charge carrying solitons will also be labelled by the charge in question.

To lowest nontrivial order the mass of this state is

$$m = E_C + \sum_i n_i \omega_i + \Delta E_0 \qquad (5)$$

where E_C is the classical soliton energy in its rest frame, ω_i is the frequency of the i'th normal mode, and ΔE_0 is the change in the zero point energy of the fields in the presence of the soliton, relative to the zero point energy in the vacuum. In those cases where it has been checked, the expression in Eq. (5) is finite after renormalization has been taken into account. This is probably the case for all renormalizable theories because ultraviolet divergences should not be affected by the presence of a slowly varying background soliton field.

This is essentially the status of the subject a year ago. Some of the more exciting recent results and offshoots are

1) In five-dimensional space-time solitons constructed of pure gauge fields have been discovered.[11] Their relevance to four dimensional space-time is the tunnelling phenomenon discussed at this conference by 't Hooft.[12]

2) The 't Hooft-Polyakov monopole carries magnetic charge. As is well known, when a magnetic charge g and an electric charge e are held near each other, the electromagnetic field surrounding them carries angular momentum $\frac{eg}{4\pi}$. By introducing into the theory charged bosons in the fundamental representation of the gauge group SU(2), one can form bound states with half integer $\frac{eg}{4\pi}$. In the quantized theory these objects should be fermions; thus, one has constructed fermions from bosons in four-dimensional space-time.[13]

3) Jackiw and Rebbi[14] have discovered a peculiar effect when fermions are coupled to topological solitons. Suppose in two-dimensional space-time we have a topological soliton with

$$\phi(x) = -\phi(-x)$$
$$\phi(+\infty) > 0 \tag{6}$$

Couple fermions to this soliton with the Lagrangian density

$$\mathcal{L} = \mathcal{L}(\phi) + i\bar{\psi}\not{\partial}\psi + g\bar{\psi}\psi\phi \tag{7}$$

We are thus led to the Dirac equation for a fermion in an odd scalar potential

$$(i\not{\partial} + \phi(x))\psi(x,t) = 0 \tag{8}$$

This equation always has a zero energy solution

$$\psi_0(x,t) = \exp\left\{- \int_0^x \phi(x')dx' \quad \psi_0(0)\right\} \tag{9}$$

where $\psi_0(0)$ satisfies

$$\psi_0(0) = i\gamma_1\psi_0(0) \tag{10}$$

This solution has zero energy but is non-degenerate. For every other solution ψ_n^+ of positive energy there exists a solution ψ_n^- of negative energy. Expanding in normal modes, we can write

$$\psi(x) = a\psi_0(x) + \sum_{n=1}^{\infty} (c_n\psi_n^+(x)+d_n^+\psi_n^-(x)) \qquad (11)$$

In the usual interpretation c_n^+ and d_n^+ create fermions and anti-fermions respectively, but the interpretation of a is not so clear. Normalize ψ_0 so that

$$[a,a^+]_+ = 1 \; . \qquad (12)$$

Jackiw and Rebbi argue that the most symmetric and simplest way to treat a is by introducing a quantum number of value $\pm\frac{1}{2}$. On these states a has the action

$$a|\tfrac{1}{2}> = |-\tfrac{1}{2}> \qquad a|-\tfrac{1}{2}> = 0$$

$$a^+|\tfrac{1}{2}> = 0 \qquad a^+|-\tfrac{1}{2}> = |\tfrac{1}{2}> \qquad (13)$$

This quantum number is then regarded as the fermion number in this zero frequency mode. Thus one has solitons of half integer fermi number. Jackiw and Rebbi show that this effect also occurs when coupling fermions to topological solitons in four dimensions, such as the monopole. The meaning of these objects in the full quantum theory is not entirely clear.

Let me conclude by listing a few of the more interesting remaining questions.

1) Is there a simple way to calculate the quantum corrections to soliton-soliton scattering? Since the classical solutions for scattering possess complicated time dependence, one must

do time dependent perturbation theory. This is in principle possible, but a simple prescription for finding the phase shifts perturbatively has not been given.

2) Does there exist a local field that can create soliton states? For the sine-Gordon theory in two dimensions such a field follows from the recently demonstrated equivalence with the massive Thirring model.[15] I suspect all quantized solitons can be created by local fields, but I have no idea how to show it.

3) Are there other non-perturbative ways to get particles out of quantum field theory? For many years conventional perturbation theory was the only extensively studied approach to quantized field theory, and this approach missed the soliton sectors. I close by noting that inverse couplings can appear in non-perturbative analyses, and we have the very intriguing relation

$$m_\pi = \frac{2}{\alpha} m_e \quad . \tag{14}$$

REFERENCES

1. T. Degrand, R.L. Jaffe, K. Johnson and J. Kiskis, Phys. Rev. D12, 2060 (1975).

2. I. Bars, Phys. Rev. Lett. 36, 1521 (1976).

3. K. Wilson, Phys. Rev. D10, 2445 (1974); J. Kogut and L. Susskind, Phys. Rev. D11, 395 (1975).

4. G. 't Hooft, Nucl. Phys. B72, 461 (1974).

5. A. Scott, F. Chu and D. McLaughlin, Proceedings IEEE 61, 1443 (1973).

6. G. 't Hooft, Nucl. Phys. B79, 276 (1974); A.M. Polyakov, JETP Lett. 20, 194 (1974).

7. J. Arafune, P. Freund and C. Goebel, J. Math. Phys. 16, 433 (1975); S. Coleman, Lectures at the 1975 International School of Subnuclear Physics "Ettore Majorana".

8. P. Vinciarelli, Lett. Nuovo Cimento 4, 905 (1972); T.D. Lee and G.C. Wick, Phys. Rev. D9, 3471 (1974); M. Creutz and K.S. Soh, ibid. 12, 443 (1975).

9. R. Friedberg, T.D. Lee and A. Sirlin, Phys. Rev. D13, 2739 (1976).

10. R. Dashen, B. Hasslacher and A. Neveu, Phys. Rev. D10, 4114 (1974); 10, 4128 (1974); 10, 4139 (1974); R. Rajaraman, Physics Reports 21, 227 (1975); J. Goldstone and R. Jackiw, Phys. Rev. D11, 1486 (1975); J.-L. Gervais and B. Sakita, Phys. Rev. D11, 2943 (1975); C.G. Callan and D.J. Gross, Nucl. Phys. B93, 29 (1975); N.H. Christ and T.D. Lee, Phys. Rev. D12, 1606 (1975); M. Creutz, Phys. Rev. D12, 3126 (1975).

11. A.A. Belavin, A.M. Polyakov, A.S. Schwartz and Y.S. Tyupkin, Phys. Lett. 59B, 85 (1975).

12. G. 't Hooft, Phys. Rev. Lett. 37, 8 (1976); C.G. Callan, R.F. Dashen and D.J. Gross (to be published); R. Jackiw and C. Rebbi, Phys. Rev. Lett. 37, 172 (1976).

13. P. Hasenfratz and G. 't Hooft, Phys. Rev. Lett. 36, 1119 (1976); R. Jackiw and C. Rebbi, Phys. Rev. Lett. 36, 1116 (1976); A.S. Goldhaber, Phys. Rev. Lett. 36, 1122 (1976).

14. R. Jackiw and C. Rebbi, Phys. Rev. D13, 3398 (1976).

15. A. Luther and V.J. Emery, Phys. Rev. Lett. 33, 589 (1974); S. Coleman, Phys. Rev. D11, 2088 (1975); S. Mandelstam, Phys. Rev. D11, 3026 (1975).