ϵ_8/ϵ_0, which is independent of the $SU_3 \otimes SU_3$ limit and is determined by the physical pseudoscalar-meson mass ratios.

After this note was submitted for publication we became aware of a work by Wada[5] in which a K_{l3} correction similar to ours is derived and used for an examination of its analytic properties. We thank H. Pagels for this information.

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*Work supported by the Deutsche Forschungsgemeinschaft.

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PHYSICAL REVIEW D VOLUME 10, NUMBER 8 15 OCTOBER 1974

Higgs mechanism and quark confinement*

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(Received 17 June 1974)

We discuss a recently proposed mechanism for quark confinement obtained by combining the Higgs mechanism for producing massive gauge mesons with the theory of magnetic monopoles. The theory is mathematically equivalent to the quarks being magnetic monopoles moving in a superconducting vacuum. When the quarks in a hadron are pulled apart, they remain connected by vortices of magnetic flux; consequently, at large interquark distances the energy of the system becomes proportional to the quark separation. At low energies the quarks are closer together than the penetration depth of the superconductor, and thus they appear effectively free.

In spite of the successes of the quark model,[1] the underlying quanta remain unobserved. This has led to a multitude of speculations on dynamical mechanisms for quark confinement.[2] In this paper we further examine a conceptually simple field-theoretical model of confinement recently proposed by Parisi.[3] In this picture the free quark does not exist.

This model is mathematically equivalent to considering the quarks as magnetic monopoles and the vacuum as a type-II superconductor. Such a superconductor can support magnetic fields over long distances only if the flux is confined within "vortex lines." When a quark-antiquark pair becomes separated by a distance large compared to the penetration depth of the superconducting vacuum, one of these vortex lines appears between them. This yields an attractive force independent of the separation and an energy proportional to the separation. When this energy becomes sufficiently large, additional quark-antiquark pairs will be produced in the vortex, thus forming a collection of magnetically neutral "mesons."

This theory has been primarily discussed in the limit in which the vortex lines are of small diameter.[4,5] In this manner one obtains the dual string model. We suggest that the model be considered when the vortex lines have radius comparable to hadronic sizes. At high excitations the model should possess the structure of the dual model, while at low energies the model will be closer to the simple quark picture.

We require a version of the quark model in which all quarks carry an unphysical conserved quantum number, often referred to as "color." This quantum number will correspond to the magnetic charge in the above analogy. The quarks combine in such a manner that the physical states have no net color. Models of this type have been described elsewhere,[1] and the detailed structure chosen is irrelevant to our discussion.

The model begins with the observation of Nielsen and Olesen[5] that the well-known Higgs[6] type of Lagrangian describes a relativistic generalization of the Ginsberg-Landau[7] phenomenological theory of superconductivity. This Lagrangian gives the superconducting vacuum discussed above. In addition to the bound quark states, the final theory contains a scalar meson and a vector meson, both remnants of the Higgs mechanism.

The starting point is the standard Higgs[6] Lagrangian

$$\mathcal{L}(x) = -\tfrac{1}{4} F_{\mu\nu}(x) F^{\mu\nu}(x) + [(\partial_\mu - ieA_\mu)\phi^*(x)][(\partial^\mu + ieA^\mu)\phi(x)] + \mu^2\phi^*(x)\phi(x) - \lambda[\phi^*(x)\phi(x)]^2 , \tag{1}$$

where

$$F_{\mu\nu}(x) \equiv \partial_\nu A_\mu(x) - \partial_\mu A_\nu(x) . \tag{2}$$

Both μ^2 and λ are positive, and e is the "electric charge" carried by the ϕ field. Although we borrow terminology from electrodynamics, it should be remembered that this is merely an analogy; in particular, e is not related to the physical charge on an electron. Because of the "wrong" sign of the mass term, $\mu^2\phi^*(x)\phi(x)$, in Eq. (1), the gauge symmetry of the Lagrangian will be spontaneously broken, and the theory is equivalent to that of a massive vector field $V_\mu(x)$ and a real scalar field $\chi(x)$ described by a Lagrangian

$$\mathcal{L}(x) = -\tfrac{1}{4} F_{\mu\nu}(x) F^{\mu\nu}(x) + \frac{e^2\mu^2}{2\lambda} V_\mu(x) V^\mu(x) + \tfrac{1}{2}[\partial_\mu\chi(x)][\partial^\mu\chi(x)] - \mu^2[\chi(x)]^2 - \tfrac{1}{4}\lambda[\chi(x)]^4 - (\lambda\mu^2)^{1/2}[\chi(x)]^3 + \tfrac{1}{2}e^2 V_\mu(x) V^\mu(x)[\chi(x)]^2 + \tfrac{1}{2}e^2(\mu^2/\lambda)^{1/2} V_\mu(x) V^\mu(x)\chi(x) \tag{3}$$

where

$$F_{\mu\nu}(x) = \partial_\nu V_\mu(x) - \partial_\mu V_\nu . \tag{4}$$

Physically, the vacuum has become filled with pairs of ϕ mesons, thereby producing a medium through which the "photon" propagates with an effective mass

$$m_V{}^2 = \frac{e^2\mu^2}{2\lambda} . \tag{5}$$

The fact that we have a massive photon is manifested in the wave equation, valid in the weak field limit (V_μ and χ small),

$$(\Box + m_V{}^2) F_{\mu\nu}(x) = 0 . \tag{6}$$

Of course, when the fields are strong, Eq. (6) must be modified by interactions.

The crucial observation is that since the field ϕ carries only electric charge, the vacuum cannot absorb magnetic flux. As a consequence, we still have the Maxwell equation

$$\vec{\nabla}\cdot\vec{B}(x) = 0 , \tag{7}$$

where $\vec{B}$ is the usual magnetic field,

$$B_i(x) = \tfrac{1}{2}\epsilon_{ijk} F^{jk}(x) = [\vec{\nabla}\times\vec{V}(x)]_i . \tag{8}$$

Here the indices i, j, and k run from 1 to 3 and ϵ_{ijk} is completely antisymmetric with $\epsilon_{123} = 1$.

Suppose we introduce into this theory a single static point source of magnetic flux, i.e., a magnetic monopole at rest. We can easily show that there is no spherically symmetric solution for $\vec{B}(x)$. If there were, Eq. (7) would require a radial $\vec{B}(x)$ obeying an inverse-square law. However, at large distances where $F_{\mu\nu}(x)$ is small, Eq. (6) requires a static $\vec{B}(x)$ to fall exponentially. We conclude that the correct solution for a monopole in this "superconducting vacuum" must spontaneously break rotational symmetry. Indeed, the field from the monopole will be concentrated into one or more "vortices" of magnetic flux running to spatial infinity. These vortices, well known in the theory of superconductivity, possess a net energy per unit length; consequently, the energy of a single monopole in an infinite superconductor is infinite.

Some properties of these vortex line solutions are easily established. Consider one such line oriented in a cylindrically symmetric fashion about the z axis. By symmetry we can write

$$\vec{B}(x_\mu) = \vec{e}_z B(r), \quad r = (x^2 + y^2)^{1/2} . \tag{9}$$

Such a form automatically satisfies Eq. (7). For weak fields Eq. (6) should be valid, yielding

$$\left[\left(\frac{d}{dr}\right)^2 + \frac{1}{r}\frac{d}{dr} - m_V{}^2\right] B(r) = 0 . \tag{10}$$

The boundary condition $B(\infty) = 0$ gives the asymptotic form

$$\ln B(r) \underset{r\to\infty}{\sim} -m_V r . \tag{11}$$

Equation (11) tells us that the vortex lines possess a characteristic radius of order $m_V{}^{-1}$. This dimension corresponds to the penetration depth of a superconductor. As r becomes small, the solution to Eq. (10) diverges logarithmically; however, this is unreliable because when the fields are strong, interactions become important and Eq. (10) breaks down. Nielsen and Olesen[5] have analyzed the vortex solution more carefully, studying also the behavior of the field χ in the vortex. As long as the mass squared of the χ meson, $m_\chi{}^2 = 2\mu^2$, is of the same order of magnitude as the vector-meson mass squared, our estimate of the vortex line size should be approximately valid.

It should now be clear that we can achieve a model of quark confinement by introducing into the above theory quarks carrying magnetic monopole moments via their color. In the literature there exist several discussions of how to formulate electrodynamics incorporating magnetic monopoles.[8] Here we only recall some of the interesting features of monopole theory. One begins with the observation that the fields $F_{\mu\nu}(x)$ around a monopole are identical to those of a semi-infinite solenoid of infinitesimal diameter ending at the monopole position. This naturally leads to the concept of a "Dirac string"; one imagines that such a solenoid produces the field of each monopole. Away from the ends, these strings should be unobservable. Generally, however, a solenoid is detectable from its exterior via the famous quantum-mechanical Aharanov-Bohm effect.[9] This possibility is evaded only for certain quantized values of the magnetic flux carried by the solenoid. Consequently, all monopole moments g_i are quantized according to the rule

$$\frac{eg_i}{2\pi}=n_i\ , \qquad (12)$$

where n_i is an integer and the index i refers to the monopole in question. Schwinger and Zwanziger have presented further arguments that for full Lorentz invariance the n_i must be chosen to be even integers. Because of the quantization condition, the usual perturbation series in the coupling constants does not exist.[8, 10]

In this theory it is important that the Dirac strings and the vortex lines not be confused. The string is purely a mathematical artifice introduced to formulate the theory, while the vortex lines are physical, observable objects. In the limit giving the dual string model it is the vortex line that becomes the dual string. It is an unfortunate historical accident that these two objects which should not be confused are both called strings.

Having presented the theory, we must discuss to what extent the quarks inside a hadron can appear free. Whenever a group of quarks with zero total color are closer together than the penetration depth m_V^{-1}, the superconductivity of the vacuum places no strong constraints on their motion. Since inelastic electron-proton scattering experiments[11] suggest a pointlike substructure for the proton at distances smaller than 1 GeV^{-1}, m_V and μ might both be expected to be of order 1 GeV, not an unreasonable value. Also, the magnetic forces at short distances should be sufficiently small that the bare quark structure survives approximately. However, we cannot take $g_i^2/4\pi$ too small if we are to avoid a sharp quasielastic peak in inelastic electron-proton scattering. Indeed, experiment suggests that the proton contains an appreciable "sea" of quark-antiquark pairs.[11] Consequently we expect

$$\frac{g_i^2}{4\pi}\lesssim 1. \qquad (13)$$

Also, a $g_i^2/4\pi$ of order unity avoids the necessity for superstrong couplings among the scalar and vector fields.

In conclusion, this model provides a simple mechanism for producing hadrons as bound states of quarks, without the single free quark state existing. In view of this simplicity, we feel that this model deserves serious consideration as a realistic description of hadrons. As in any model, several interesting questions remain. What are the mesons corresponding to the V_μ and χ fields? To what extent do these mesons mix with singlet bound quark states? How much can the model gain by consideration of non-Abelian gauge theories in the Higgs mechanism? At what energies does one probe the interior of the quarks themselves? Can we define a meaningful bare quark mass? Indeed, the model may pose more questions than it answers.

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*Work performed under the auspices of the U. S. Atomic Energy Commission.

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PHYSICAL REVIEW D VOLUME 10, NUMBER 8 15 OCTOBER 1974

Vacuum polarization in homogeneous magnetic fields*

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(Received 24 June 1974)

The lowest-order vacuum polarization in homogeneous magnetic fields is calculated exactly, using a "momentum" representation of the electron Green's function obtained by Schwinger in 1951.

In quantum electrodynamics, the electron and photon propagation functions, $G(x', x'')$ and $D_+(x'-x'')$, are the building stones in calculating higher-order processes. In free space, we know that it is more convenient to express them in the momentum form

$$G(p)=\frac{1}{m+\gamma p-i\epsilon}, \tag{1}$$

$$D_+(k)=\frac{1}{k^2-i\epsilon}. \tag{2}$$

In the presence of constant external fields, D_+ remains the same while G satisfies the differential equation

$$\left[m+\gamma\left(\frac{1}{i}\partial'-eA(x')\right)\right]G(x',x'')=\delta(x'-x''), \tag{3}$$

which can be solved exactly, by using the proper-time method.[1] After separating the gauge-dependent part, the remaining gauge-independent part can be easily cast in the "momentum" representation. For the purely homogeneous magnetic field case (with $F_{12}=-F_{21}=H$), the result[1,2] is

$$G(x',x'')=\Phi(x',x'')\int\frac{(dp)}{(2\pi)^4}e^{ip(x'-x'')}\mathcal{G}(p), \tag{4}$$

where

$$\mathcal{G}(p)=i\int_0^\infty ds\,\exp\left[-is\left(m^2+p_{\parallel}^2+\frac{\tan z}{z}p_\perp^2\right)\right]\times\left[(m-\gamma p_\parallel)e^{i\sigma_3 z}-\frac{1}{\cos z}\gamma p_\perp\right]\frac{1}{\cos z}, \tag{5}$$

$$\Phi(x',x'')=\exp\left[ie\int_{x''}^{x'}A(\xi)d\xi\right], \tag{6}$$

$$(ab)_\parallel=-a^0b^0+a_3b_3,\quad (ab)_\perp=a_1b_1+a_2b_2, \qquad z=seH. \tag{7}$$

The combination of Eqs. (2) and (4) can be used to compute higher-order processes in the homogeneous magnetic field. In a previous note,[2] we calculated the electron mass operator

$$M(x',x'')=ie^2\gamma^\mu G(x',x'')D_+(x'-x'')\gamma_\mu+\text{c.t.} \tag{8}$$

by using this technique. Here we wish to apply the same method to calculate the vacuum-polarization process in a homogeneous magnetic field.[3]

The Lagrangian corresponding to the lowest-order vacuum-polarization process is

$$\mathfrak{L}^{(2)}=ie^2\int(dx')(dx'')\,\mathrm{Tr}\,\gamma A_1(x')G(x',x'')\gamma A_2(x'')\times G(x'',x')+\text{c.t.}, \tag{9}$$

where the contact terms (c.t.), which will be determined later, are designed to satisfy the normalization condition and gauge-invariant requirement. In calculating Eq. (9), instead of using the conventional regularization scheme while carefully maintaining the gauge-invariant requirement, we will use the noncausal method of source theory.[4] Substituting Eq. (4) into Eq. (9), defining

$$A(x)=\int\frac{(dk)}{(2\pi)^2}e^{ikx}A(k), \tag{10}$$

and noting that

$$\Phi(x',x'')\Phi(x'',x')=1, \tag{11}$$