

1303 (1972).

<sup>42</sup>A comparison of the amplitudes to only the differential cross section cannot distinguish the sign of  $\text{Re}M_{\Delta\lambda=0}$  for  $-t > 0.2 \text{ GeV}^2$  between the present solution, Fig. 4(b), and a solution similar to the  $\rho$  amplitude, Fig. 4(a). If the  $\eta^0$  polarization is calculated assuming  $\text{Re}M_{\Delta\lambda=0} < 0$  for  $-t < 1.0 \text{ GeV}^2$ , the resulting polarization prediction is qualitatively similar to the curve in Fig. 3 but has a smaller magnitude in the region  $-t > 0.4 \text{ GeV}^2$ .

<sup>43</sup>At  $5 \text{ GeV}/c$ , typical values of  $A'$  and  $r$  are  $1.3 \text{ GeV}^{-2}$  and  $5 \text{ GeV}^{-1}$ , respectively. With these parameters the difference between  $\Delta\lambda=0$  and  $\Delta\lambda=1$  amplitudes is then  $< 10\%$  for  $b > 2.6 \text{ GeV}^{-1}$  ( $\sim \frac{1}{2} \text{ fm}$ ).

<sup>44</sup>A. Firestone, G. Goldhaber, A. Hirata, D. Lissauer, and G. H. Trilling, *Phys. Rev. Letters* **25**, 958 (1970).

<sup>45</sup>F. J. Gilman, *Phys. Rev.* **171**, 1453 (1968).

<sup>46</sup>Wherever possible the quantities shown in Figs. 8–11 are taken from data in the region  $0.0 \leq -t \leq 0.4 \text{ GeV}^2$ . For the reaction  $\bar{K}N \rightarrow \pi\Lambda$ , there is no clear break in the differential cross section, and slopes were accepted when calculated over a large momentum-transfer range.

<sup>47</sup>For example, see Table III where it is seen that an appropriate choice of SU(3)  $f/d$  ratios could account for the observed mirror symmetry.

<sup>48</sup>In  $\Sigma$  production isospin- $\frac{3}{2}$  states can also contribute in the  $t$  channel. Such exchanges are exotic, and are expected to decrease in importance very rapidly with energy. Experimental tests made above  $3 \text{ GeV}/c$  indicate predominance of isospin  $\frac{1}{2}$  in the  $t$  channel [see D. J. Crennell, H. A. Gordon, K. W. Lai, and J. M. Scarr, *Phys. Rev. D* **6**, 1220 (1972); L. Moscoso, J. R. Hubbard, A. Laveque, J. P. de Brion, C. Louedec, D. Revel, J. Badier, E. Barrelet, A. Rouge, H. Videau, and I. Videau, *Nucl. Phys.* **B36**, 332 (1972)].

<sup>49</sup>M. Ferro-Luzzi, H. K. Shepard, A. Vernan, R. T. Poe, and B. C. Shen, *Phys. Letters* **34B**, 524 (1971);

A. Kernan, R. T. Poe, B. C. Shen, I. Butterworth, M. Ferro-Luzzi, and H. K. Shepard, U. C. Riverside report, 1972 (unpublished).

<sup>50</sup>A recent analysis (Ref. 20) of the  $\Sigma$  and  $\Lambda$  reactions does assume strong exchange degeneracy for the helicity-flip amplitudes and consequently implies either nonperipheral imaginary parts for the nonflip amplitudes, or substantially different radii of interaction for  $K^*$  and  $K^{**}$  amplitudes.

<sup>51</sup>A. Bashian, G. Finocchiaro, M. L. Good, P. D. Gramis, O. Guisan, J. Kirz, Y. Y. Lee, R. Pittman, G. C. Fischer, and D. D. Reeder, *Phys. Rev. D* **4**, 2667 (1971).

<sup>52</sup>In separate comparisons of the DAM amplitudes to the  $\Sigma$  and  $\Lambda$  data values were obtained of  $\Delta\alpha = 0.12$  and  $\Delta\alpha = 0.01$ , respectively. For the comparisons tabulated in Table V and shown in the figures the parameter  $\Delta\alpha$  was fixed to the average of the  $\Sigma$  and  $\Lambda$  results to simplify the evaluation of SU(3)  $f/d$  factors. We have used  $\alpha(t) = 0.5 + 0.9t$  and  $\alpha_V(0) = 0.33$ .

<sup>53</sup>We note that the precise values of these ratios are significantly correlated to the choice for the phase of the amplitudes at  $t=0$ , which unfortunately is only poorly determined by the present data. This correlation is mainly due to the approximate equality,  $\text{Re}A_V \approx \text{Re}A_T$  [see discussion following Eq. (16)], which implies the following ratio for  $K^*$  and  $K^{**}$  coupling constants [see Eqs. (8), (9), and (17)]:  $g^V/g^T \sim \cot[\frac{1}{2}\pi\alpha_V(0)]^2$ . This ratio is  $\sim 3$  for the  $K^*$  trajectory intercept,  $\alpha_V(0) = 0.33$ , used in the present analysis. However, if we choose the  $\rho$  Regge intercept,  $\alpha_V(0) = 0.5$ , we then find the ratio  $g^V/g^T \sim 1$  and the ratio  $\text{Im}M_{\Delta\lambda=0}/\text{Re}M_{\Delta\lambda=0} \sim 0.4$  for both  $\Sigma$  and  $\Lambda$  channels. The amplitudes resulting from the use of the  $\rho$  Regge phase yield polarization and differential cross-section predictions that cannot be excluded by the present data.

## Low-Energy Theorems and High-Energy Behavior\*

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We study the constraints at high energy that analyticity imposes on amplitudes given a low-energy theorem and an upper bound to the amplitude at intermediate energies. These constraints prove to be nontrivial when applied to the pion electromagnetic form factor. We suggest that our results may be useful in a future analysis of the contribution of the spin-dependent forward Compton amplitude to unpolarized Compton scattering. This application will require experimental data on the forward differential cross section in the resonance region.

In a recent paper Levin, Mathur, and Okubo<sup>1</sup> have phenomenologically discussed bounds on the pion charge radius in terms of the modulus of the pion electromagnetic form factor on its cut. In this analysis they discovered an interesting side

result. They found that the modulus of the pion form factor cannot begin to fall rapidly in a "dipole" fashion until a four-momentum transfer squared of at least  $17 \text{ GeV}^2$  is reached. What is remarkable about this result is the small amount

of physical input that went into it; they used (1) analyticity, (2) the low-energy theorem  $F_\pi(0)=1$ , (3) experimental data on the modulus of the form factor for momentum transfers squared of less than  $4.4 \text{ GeV}^2$ , and (4) a smooth continuation of the experimental data up to some momentum transfer where a "dipole" behavior begins. This result strikingly illustrates that analyticity imposes constraints on high-energy behavior in terms of the low-energy properties of an amplitude.

The purpose of this paper is to make these constraints of analyticity more precise. We consider an amplitude which is a real analytic function of an energy variable except for possible cuts on the real axis. In this variable we assume that the amplitude satisfies a low-energy theorem. We further assume that below a certain energy on the cuts we have an upper bound on the modulus of the amplitude. We then find rigorous bounds on the amplitude above this energy. These bounds are the best possible in the sense that we can find analytic functions that obey our conditions and saturate the bounds.

In addition to the pion form factor, we consider the application of our results to forward Compton scattering. Here low-energy theorems give the amplitudes at zero photon energy. We conclude that our result is not very strong on the spin-independent part of the Compton amplitude. However, we find interesting lower bounds on the spin-dependent part of the amplitude and its contribution to unpolarized forward Compton scattering.

We now pose the mathematical problem. We assume that  $F(t)$  is a real analytic function of  $t$  in the entire  $t$  plane except for a cut on the positive real axis for  $t$  in the range  $+\infty > t \geq t_0 > 0$ . We take

$F(t)$  to be polynomially bounded at infinity and to have no essential singularities on the cut. We further suppose that  $F(t)$  satisfies a low-energy theorem which, with no essential loss of generality, we take to be  $F(0)=1$ .

Given such an amplitude  $F(t)$ , we now presume that there is an experimental upper bound on  $|F(t)|^2$  for  $t$  on the cut below some arbitrary value of  $t$ . More precisely, we assume

$$|F(t)|^2 \leq H(t) \text{ for } t_0 \leq t \leq N. \quad (1)$$

We now consider an arbitrary positive and polynomially bounded function  $w(t)$  defined on  $t \geq N$ . What we shall find is a lower bound on the quantity

$$\max_{t \geq N} w(t) |F(t)|^2 \quad (2)$$

in terms of  $H(t)$  and  $w(t)$ . Our lower bound is the best lower bound in the sense that we can find a function satisfying the given properties of  $F(t)$  and saturating this bound. As would be expected, this bound becomes more stringent as  $H(t)$  is reduced toward  $|F(t)|^2$ .

To find the desired bound, we make use of the inequality

$$\int_{t_0}^{\infty} \frac{dt \ln |F(t)|^2}{t(t-t_0)^{1/2}} \geq 0. \quad (3)$$

Up to a numerical coefficient, the left-hand side of this is just the parameter  $\epsilon$  of Ref. 1. Inequality (3) is a generalization of the maximum-modulus theorem and is proven in Refs. 2 and 3. Any analytic function satisfying our assumptions must obey this condition. From inequality (3) we quickly find our lower bound on expression (2):

$$\begin{aligned} 0 &\leq \int_{t_0}^{\infty} \frac{dt \ln |F(t)|^2}{t(t-t_0)^{1/2}} = \int_{t_0}^N \frac{dt \ln |F(t)|^2}{t(t-t_0)^{1/2}} + \int_N^{\infty} \frac{dt \ln [w(t) |F(t)|^2]}{t(t-t_0)^{1/2}} - \int_N^{\infty} \frac{dt \ln w(t)}{t(t-t_0)^{1/2}} \\ &\leq \int_{t_0}^N \frac{dt \ln H(t)}{t(t-t_0)^{1/2}} + \ln \left\{ \max_{t \geq N} [w(t) |F(t)|^2] \right\} \int_N^{\infty} \frac{dt}{t(t-t_0)^{1/2}} - \int_N^{\infty} \frac{dt \ln w(t)}{t(t-t_0)^{1/2}}. \end{aligned} \quad (4)$$

Solving for the expression of interest gives

$$\max_{t \geq N} [w(t) |F(t)|^2] \geq \exp \left[ \left( - \int_{t_0}^N \frac{dt \ln H(t)}{t(t-t_0)^{1/2}} + \int_N^{\infty} \frac{dt \ln w(t)}{t(t-t_0)^{1/2}} \right) / \int_N^{\infty} \frac{dt}{t(t-t_0)^{1/2}} \right]. \quad (5)$$

In the Appendix we demonstrate that this is the best possible bound by giving a function that saturates it. Inequality (5) is our main result.

We can simplify this bound by taking various limits. If we take  $N$  to  $t_0$ , we get

$$\max_{t > t_0} [w(t) |F(t)|^2] \geq \exp \left[ \frac{t_0^{1/2}}{\pi} \int_{t_0}^{\infty} \frac{dt \ln w(t)}{t(t-t_0)^{1/2}} \right]. \quad (6)$$

Another interesting case is obtained by letting  $N$  be much larger than  $t_0$  and taking  $w(t) = t^\alpha$ . This

gives the result

$$\max_{t \geq N \gg t_0} \left[ \left( \frac{t}{2N} \right)^\alpha |F(t)|^2 \right] \geq \exp \left[ -\frac{1}{2} N^{1/2} \int_{t_0}^N \frac{dt \ln H(t)}{t(t-t_0)^{1/2}} \right]. \quad (7)$$

It is this form that we use the most in the following discussion.

Finite-energy sum rules<sup>4</sup> (FESR) have for several years been exploited as a method of relating high- and low-energy behaviors of scattering amplitudes. In that program strong assumptions are made on Regge asymptotic behavior with only a few dominant trajectories. In this paper we make no such assumptions and thus our results are on a more rigorous footing. It is interesting to compare our results to those of FESR. Qualitatively, the smaller  $H(t)$  is for  $t \leq N$ , the larger is our lower bound involving  $|F(t)|^2$  for  $t \geq N$ . FESR suggest that a small low-energy amplitude is associated with small high-energy behavior. This contrast is not actually a conflict because we impose the additional constraint  $F(0)=1$ .

In this paper we give lower bounds on an amplitude. One might ask whether there exist upper bounds as well. On the basis of our input, the answer to this question is no. This is essentially because a zero could occur close to the point of the low-energy theorem, thereby allowing the amplitude elsewhere to be as large as desired.

As a first application, let us sketch how the Levin, Mathur, and Okubo condition<sup>1</sup> on the large-momentum-transfer behavior of the pion form factor follows from our bound. Taking  $F(t)$  to be the pion form factor, we assume  $|F(t)| \propto 1/t^2$  for  $t \geq N$ . If we let  $w(t) = t^4$ , we find a bound on the coefficient of the  $t^{-4}$  behavior of  $|F(t)|^2$  in terms of the form factor for  $t$  below  $N$ . If, following Ref. 1, we force a smooth connection between their extrapolation of the data below  $N$  and the dipole behavior above  $N$ , then their condition on  $N$  follows. This constraint on  $N$  depends strongly on the assumed form factor (or an upper bound to the form factor) below  $N$ . A presently unknown  $J=1$ ,  $C=-$  resonance above 4.4 GeV<sup>2</sup> can invalidate this result.

We now apply our bounds to Compton scattering from a spin- $\frac{1}{2}$  target. Our notation follows Damashek and Gilman.<sup>5</sup> In terms of the Pauli spinors for the target particle in its initial and final states, the forward Compton amplitude can be written

$$f(\nu) = \chi_f^* [f_1(\nu) \vec{\epsilon}_2^* \cdot \vec{\epsilon}_1 + i \vec{\sigma} \cdot (\vec{\epsilon}_2^* \times \vec{\epsilon}_1)] f_2(\nu) \chi_i, \quad (8)$$

where  $\nu$  is the lab photon energy and  $\vec{\epsilon}_1(\vec{\epsilon}_2)$  is the polarization vector for the initial (final) photon. We normalize the amplitude such that

$$\left. \frac{d\sigma}{d\Omega} \right|_{0^\circ, \text{lab}} = |f(\nu)|^2. \quad (9)$$

Both  $f_1(\nu)$  and  $f_2(\nu)$  can be continued as analytic functions in the cut  $\nu$  plane with cuts from  $\nu_0$  to  $\infty$  and  $-\infty$  to  $-\nu_0$ , where  $\nu_0$  is the inelastic threshold energy. We work to lowest nonvanishing order in the electric charge so that  $\nu_0$  is nonzero. In this analytic continuation,  $f_1(\nu)$  is an even function of  $\nu$  and  $f_2(\nu)$  is odd. The amplitudes  $f_1(\nu)$  and  $f_2(\nu)$  satisfy the low-energy theorems

$$\begin{aligned} f_1(0) &= -\frac{\alpha Q^2}{m}, \\ f_2'(0) &= -\frac{\alpha \kappa^2}{2m^2}, \end{aligned} \quad (10)$$

where  $\alpha = \frac{1}{137}$ ,  $Q$  is the target charge in units of the electron charge,  $\kappa$  is the target anomalous magnetic moment, and  $m$  is the target mass.

We first apply our bound to the amplitude  $f_1(\nu)$ . Since  $f_1(\nu)$  is an even function of  $\nu$ , we consider it as a function of  $\nu^2$  with only one cut in the  $\nu^2$  plane from  $\nu_0^2$  to  $\infty$ . In our general discussion  $t$  should now be interpreted as  $\nu^2$ . To choose a useful  $w(\nu^2)$ , we appeal to Regge theory. At high energies the amplitude  $f_1(\nu)$  should be dominated by Pomeranchukon exchange. This gives asymptotically

$$f_1(\nu) \sim i c \nu, \quad (11)$$

suggesting that we pick

$$w(\nu^2) = \nu^{-2}. \quad (12)$$

Using Eq. (7), we get

$$\max_{\nu \geq N \gg \nu_0} \left( \frac{e^2 N^2}{\nu^2} \frac{|f_1(\nu)|^2}{|f_1(0)|^2} \right) \geq \exp \left[ -N \int_{\nu_0}^N \frac{d\nu \ln H(\nu)}{\nu(\nu^2 - \nu_0^2)^{1/2}} \right], \quad (13)$$

where  $H(\nu)$  is any upper bound to  $|f_1(\nu)|^2/|f_1(0)|^2$  for  $\nu$  between  $\nu_0$  and  $N$ . Of course  $H(\nu)$  can be replaced by  $|f_1(\nu)|^2/|f_1(0)|^2$  where it is known.

It is easy to convince oneself that experimentally this is a rather weak restriction on  $f_1(\nu)$  when the target is a proton. This is because experiment indicates that  $|f_1(\nu)|$  is quite large.<sup>5</sup> Already at a photon lab energy of only 0.22 GeV the imaginary part of  $f_1(\nu)$  exceeds  $f_1(0)$  in magnitude. Since our only input was analytic properties and the low-energy theorem, a dispersion relation calculation of  $f_1(\nu)$  using the low-energy theorem will automatically satisfy inequality (11). This means that  $f_1(\nu)$  as given by Damashek and Gilman<sup>5</sup> must obey our constraint. This amplitude adequately describes all presently available data.

Let us comment that although our bound in this case appears quite weak, it is likely that inequality (3) is in actuality an equality. This would be the case if, and only if,  $f(\nu)$  has no zeros in the entire  $\nu$  plane away from the cuts.<sup>2,3</sup> The analysis of Ref.

5 indicates that  $f_1(0)$  has no zeros for real  $\nu$ . But  $f_1(\nu)$  can have zeros only for real  $\nu$ , as can be verified from positivity, the usual dispersion relation, and the fact that  $f_1(0)$  is nonpositive. For inequality (3) to become an equality requires a quite small  $|f_1(\nu)|$  near threshold in order to cancel the positive contributions to the integral from larger  $\nu$ . This is what happens with the amplitude given in Ref. 5.

We now turn our attention to the amplitude  $f_2(\nu)$ . Since  $f_2(\nu)$  is an odd function, we consider

$$F(\nu^2) = \frac{f_2(\nu)}{\nu f_2'(0)}. \quad (14)$$

This function should satisfy our assumptions on  $F(t)$ . To pick  $w(t)$  we again appeal to Regge theory. The asymptotic behavior of  $f_2(\nu)$  is controlled by exchanges of abnormal parity with positive charge conjugation and odd signature. The highest such known trajectory is that of the  $A_1$  meson with  $\alpha_{A_1}(0) \approx 0$ . This gives the asymptotic behavior

$$f_2(\nu) \sim i \times \text{const}. \quad (15)$$

Therefore we have

$$|F(\nu^2)|^2 \sim \frac{\text{const}}{\nu^2}. \quad (16)$$

This suggests choosing

$$w(\nu^2) = \nu^2. \quad (17)$$

Using inequality (7), we get

$$\max_{\nu \geq N \gg \nu_0} |f_2(\nu)|^2 \geq e^{2N^2} [f_2'(0)]^2 \exp\left(-N \int_{\nu_0}^N \frac{d\nu \ln H(\nu)}{\nu(\nu^2 - \nu_0^2)^{1/2}}\right), \quad (18)$$

where  $H(\nu)$  is any function satisfying

$$H(\nu) \geq \frac{|f_2(\nu)|^2}{\nu^2 [f_2'(0)]^2} \text{ for } \nu_0 \leq \nu \leq N. \quad (19)$$

Inequality (18) is a nontrivial restriction imposed by analyticity on the high-energy contribution of  $|f_2(\nu)|^2$  to forward Compton scattering.

Let us remark that  $|f_2(\nu)|^2$  can be determined from unpolarized cross sections. This follows from the fact that the unpolarized forward Compton cross section is given by

$$\frac{d\sigma}{d\Omega} \Big|_{0^\circ, \text{lab}} = |f_1(\nu)|^2 + |f_2(\nu)|^2, \quad (20)$$

while  $f_1(\nu)$  is well determined from unpolarized total cross sections through the use of dispersion relations. Thus inequality (18) gives a nonzero lower bound on the experimentally measurable  $f_2$  contribution to unpolarized Compton scattering.

Present experimental data on the forward cross section are quite limited. For  $\nu$  of 4 to 17 GeV,  $|f_1(\nu)|^2$  essentially explains the experiments on Compton scattering from protons, implying that  $|f_2(\nu)|^2$  contributes less than 10% of the cross section.<sup>6</sup> This means that in this range

$$|f_2(\nu)|^2 \lesssim 2 \times 10^{-32} \left(\frac{\nu}{m}\right)^2 \text{ cm}^2. \quad (21)$$

There are presently no direct data in the resonance region, where  $|f_2(\nu)|^2$  is expected to be largest. The isobar model used by Drell and Hearn<sup>7</sup> indicates that at the  $\Delta(1236)$  resonance

$$|f_2(\nu_R)|^2 \approx 1.3 \times 10^{-31} \text{ cm}^2. \quad (22)$$

Using our bound in the form of Eq. (6), we get

$$\max_{\nu \geq \nu_0} |f_2(\nu)|^2 \geq 6.2 \times 10^{-33} \text{ cm}^2. \quad (23)$$

This is considerably below the value in Eq. (22). Consequently, in order for our bound to be useful the resonances should be removed. This is most easily accomplished by setting  $N$  at an energy above the resonance region, although it could also be done with a properly chosen  $w(\nu^2)$ . These procedures are essentially equivalent. Our bounds depend critically on the low-energy data; thus, further experiments in the resonance region are needed to determine their utility. Of course any resonance model that is analytic and satisfies the low-energy theorem will automatically satisfy our bounds.

In summary, we have found lower bounds that analyticity imposes on amplitudes at high energy in terms of low-energy theorems and intermediate-energy data. This constraint when applied to the pion form factor has shown itself to be nontrivial when confronted with experimental data. We indicate that the application to the amplitude  $f_2$  of forward Compton scattering may also be interesting. This awaits further experimental information. Our result does not appear to be useful on  $f_1$  of forward Compton scattering because this amplitude is experimentally quite large compared to its low-energy value.

#### APPENDIX

Here we give a function that satisfies all our conditions on  $F(t)$  and saturates the bound in inequality (5):

$$F(t) = \exp\left\{\frac{(t_0 - t)^{1/2}}{2\pi} \left[ \int_{t_0}^N \frac{dt' \ln H(t')}{(t' - t)(t' - t_0)^{1/2}} - \int_N^\infty \frac{dt' \ln w(t')}{(t' - t)(t' - t_0)^{1/2}} \right]\right\}$$

$$+ \left( \int_N^{\infty} \frac{dt'}{(t'-t)(t'-t_0)^{1/2}} / \int_N^{\infty} \frac{dt'}{t'(t'-t_0)^{1/2}} \right) \left( - \int_{t_0}^N \frac{dt' \ln H(t')}{t'(t'-t_0)^{1/2}} + \int_N^{\infty} \frac{dt' \ln w(t')}{t'(t'-t_0)^{1/2}} \right) \}. \quad (\text{A1})$$

We take the branch of the square-root function with positive real part where it occurs in this equation.

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## Test for Fractionally Charged Partons from Deep-Inelastic Bremsstrahlung in the Scaling Region\*

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We show that measurements of deep-inelastic bremsstrahlung,  $e^\pm + p \rightarrow e^\pm + \gamma + \text{anything}$ , in the appropriate scaling region will provide a definitive test for fractionally charged constituents in the proton, provided the parton model is valid. More precisely, measurement of the difference between the scaling inclusive bremsstrahlung cross sections of the positron and electron will allow the determination of a proton structure function  $V(x)$  which, unlike the deep-inelastic  $e-p$  structure functions, obeys an exact sum rule based on conserved quantum numbers. In particular, we show that  $\int_0^1 dx V(x) = \frac{1}{3}Q + \frac{2}{9}B (= \frac{5}{9}$  for a proton target) in the quark model, whereas  $\int_0^1 dx V(x) = Q$  in the case of integrally charged constituents. Since the result is independent of the momentum distribution of the partons, the sum rule holds for nuclear targets as well. Since  $V(x)$ , which involves the cube of the parton charge, is related to odd-charge-conjugation exchange in the  $t$  channel, Pomeron, and other  $C$ -even contributions are not present, so that  $V(x)$  should have a readily integrable quasielastic peak. This, combined with the fact that there exists a simple kinematic region in which the difference is of the same order as the inclusive bremsstrahlung cross sections themselves, and the fact that there is no hadronic-decay background, should make this a feasible experiment on proton and nuclear targets.

### INTRODUCTION

The observation of scaling in the highly inelastic limit of electron-proton scattering has excited considerable interest in constituent models of hadrons. The existence of charged, structureless "partons" in the nucleon, together with an assumption limiting the partons' momentum distribution, is sufficient to derive scaling.<sup>1</sup> It is also well known that to account for scaling it is not necessary to postulate the full apparatus of a parton model but instead only to abstract from such a theory the singular behavior of current commutators in the vicinity

of the light cone.<sup>2</sup>

Since they are more specific, however, parton models make concrete predictions which cannot be obtained from more general light-cone considerations. An example is the prediction of scaling in the process  $p + p \rightarrow \mu^+ + \mu^- + \text{anything}$ <sup>3</sup> at high energy and large  $(\mu^+ \mu^-)$  invariant mass. A test of this prediction will be central in establishing the parton model independently of the light-cone approach.<sup>4</sup> More recently the parton model has been found to provide a particularly simple explanation of large-angle exclusive scattering.<sup>5</sup> Although the parton model may be only an abstraction of a more com-