

When time goes sideways

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Abstract

General relativity deeply entwines time and space. Formulating dynamics requires a convention for time. As one enters a black-hole, time and space interchange places. With a metric that removes the singularity at the black hole horizon, one can cover space-time with a series of space-like surfaces that never reach the singularity at the black hole center. On such a set of surfaces no information is ever lost, although on black hole evaporation the space-like surfaces become disconnected.

Physics describes what things are made of and how they evolve. But the word “evolve” requires a concept of “time.” For the non-relativistic situation, either classically or quantum mechanically, time is simply an external variable.

Things get more interesting when special relativity comes into play. Then the concept of “time” is different for observers moving with relative velocities. For an observer moving at some velocity v in say the z direction relative to another observer at rest, we have the famous Lorentz transformation of coordinates

$$t' = \frac{t - vz/c^2}{\sqrt{1 - v^2/c^2}}, \quad z' = \frac{z - vt}{\sqrt{1 - v^2/c^2}}, \quad x' = x, \quad y' = y. \quad (1)$$

Here c is the speed of light, $\{t', x', y', z'\}$ are the time and space coordinates for the moving observer, and $\{t, x, y, z\}$ are for the stationary one.

Classically it is well known how to set up dynamics consistent with this transformation being a symmetry. Quantum mechanically we have relativistic quantum field theory, which introduces additional consequences. In particular, antiparticles must appear in order to maintain the impossibility of sending information faster than the speed of light. Another subtle consequence is the connection between spin and statistics: half integer spin particles must be fermions, while whole integer ones are bosons.

The above Lorentz transformation preserves a “distance” between two nearby space-time points

$$ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2 = c^2 dt'^2 - dx'^2 - dy'^2 - dz'^2. \quad (2)$$

Here $\{dt, dx, dy, dz\}$ represents the distance (usually considered as infinitesimal) between two space-time points for the corresponding observers. Let us now simplify this slightly by working in units where $c = 1$.¹ Thus this becomes

$$ds^2 = dt^2 - d\vec{x}^2. \quad (3)$$

Now where things get really interesting is in general relativity, where time depends not just on how fast one moves, but also where one is located. Clocks on top of a mountain run faster than those at sea level. This is a very small effect, but needs to be taken into consideration when precise timing is necessary, such as for GPS systems.

This effect is manifest in the famous Schwarzschild formula generalizing the above metric to the space outside a stationary and non-rotating gravitating object [1]

$$ds^2 = (1 - 1/r)dt^2 - (1 - 1/r)^{-1}dr^2 - r^2 d\Omega^2 \quad (4)$$

Here we have switched to polar coordinates for the spatial components

$$x = r \sin(\theta) \cos(\phi), \quad y = r \sin(\theta) \sin(\phi), \quad z = r \cos(\theta)$$

¹Approximately true if we measure distances in feet and time in nanoseconds.

and $d\Omega^2 = d\theta^2 + \sin^2(\theta)d\phi^2$. For simplicity, measure distances in units of the “Schwarzschild radius” for the object. For a black hole the radius of the horizon is taken as unity.

Light cones, surfaces covered by following $ds^2 = 0$ curves, behave peculiarly as one approaches the horizon at $r = 1$. They “squish” together as one moves in from outside. At the horizon, light can no longer escape. And continuing through the horizon the concepts of space-like and time-like effectively change places, with the coefficients of both dt^2 and dr^2 exchanging sign.

To appreciate the peculiar nature of these coordinates, consider an ingoing light wave approaching the black hole radially from some initial point t_0, r_0 . On this trajectory, the vanishing of the metric tells us

$$dt = -\frac{r}{r-1} dr \quad (5)$$

which is easily integrated to give

$$t - t_0 = r - r_0 - \log\left(\frac{r-1}{r_0-1}\right). \quad (6)$$

We see that as r approaches unity, time goes to infinity! The light never reaches the horizon. With the Schwarzschild metric, not only can light not leave the black hole, it can’t get in either. And as nothing goes faster than light, nothing else can reach the horizon in finite time.

As is well known, the singularity displayed in the Schwarzschild metric at $r = 1$ is somewhat artificial. It is possible to smooth this out with an appropriate redefinition of coordinates. This was first made clear by Lemaitre [2]. This raises a question. Can we find a remapping of our coordinates so that time-like surfaces bend backward and only reach the point where $r = 0$ at infinite time? If we can do this, setting up a dynamics on these surfaces would avoid any paradoxes concerning information loss as objects enter a black hole. They would always be somewhere on every time slice, although of course they may not be able to communicate with observers outside. The idea of space-time regions being unable to communicate is already familiar in special relativity, where no communication can occur between space-like separated points.

To proceed, consider a new definition of time w using a change of variables similar to that made by Finkelstein [3]

$$w = t + \log(1 - 1/r). \quad (7)$$

The addition is singular at the horizon, $r = 1$, but this singularity serves to smooth out the singularity in the Schwarzschild metric. Also inside the black hole this transformation becomes complex, but this again comes from removing the singularity.

Using this new time the metric equation reduces to

$$ds^2 = dw^2 (1 - 1/r) - 2dw dr/r^2 - dr^2 (1 + 1/r)(1 + 1/r^2) - r^2 d\Omega^2. \quad (8)$$

This is a smooth real function as one goes through the horizon. The off-diagonal term proportional to $dw dr$ rotates the “time-like” w direction

with space. In the process constant w surfaces continue to have negative ds^2 ; *i.e.* they remain space-like. Light cones turn gradually inwards until at the horizon they effectively go “sideways,” with both ingoing and outgoing light waves flowing towards the origin. Note that a singularity remains at $r = 0$. However, this occurs at infinite values of the new time w . In the new coordinates the origin is never reached.

One of the complications of trying to do quantum field theory in curved space is that a stable ground state does not exist [4]. In general, Hawking radiation occurs from non-flat regions, with the wavelength of the radiation related to the scale of the curvature. One of the dramatic consequences is that black holes, at least in the conventional definition of time, eventually evaporate. At some point in space-time a black hole will cease to exist. How does this process appear in our new coordinates? Effectively, space-like surfaces split into disconnected pieces. There is the smooth ordinary space and time after the evaporation, but the space-like surfaces inside the black hole are still there, albeit at earlier Schwarzschild times. Any information tossed into the black hole is still there when we consider time evolution in w rather than the original t variable.

We conclude that to study dynamics in physical situations with an event horizon, it is natural to define time in a way that goes sideways relative to conventional variables inside the horizon. This can be done in a manner that the origin of the black hole occurs at infinite time. On black hole evaporation, space-like surfaces become disconnected, but all information from earlier events remains somewhere.

Acknowledgement

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